Low-frequency Electromagnetic Field Analysis

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Abstract :

Among the tidal wave of innovation that has come to be symbolized by electric cars and wind power generators, a significant transition to high efficiency is being sought after in so-called mature models of electrical instruments such as motors and transformers, as well. In order to respond to this demand, it is effective to draw upon the strengths of analysis technologies like FEA and make achievements in electromagnetic field analysis.

From the standpoint of the utilization rate in electromagnetic field analysis design, however, it seems like it has not yet caught up with structural analysis. In the structural analysis field, which has more than a ten year head start on electromagnetic field analysis, is already commercially viable, and has over ten times the number of users, they use a better FEA and have incorporated it "naturally" into product designs. It is also continuing to evolve. Is there really nothing to be learned from this? Is electromagnetic field analysis evolving?

There is no doubt that electromagnetic field analysis will become an essential design technology in the future, as well. Or rather, it has to. In order for this to happen, those who are engaged in electromagnetic field analysis must grapple with new challenges without becoming satisfied with the technology and usage methods currently available.

In addition, among future analysis technology, multiphysics analysis is a vital issue, and analysis technicians cannot afford to shut themselves up in their own analysis specialties. It is necessary to actively understand the technology and culture of each other's fields.

In this presentation I will compare electromagnetic field analysis with structural analysis and talk about what kind of simulation technology is necessary for electromagnetic field analysis to play more of an active role in product designs in the future, and the accompanying challenges in usage techniques. I will also present the direction of CAE as a whole along with the latest research results.

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and
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OUTLINE

(1) Low-frequency Electromagnetic Field Analysis

(2) Magnetic Force/Torque Calculation

(3) Structural Topology Optimization

1. Magnetic Actuator
2. Inductor
3. Switched Reluctance Motor

(4) Conclusion

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Maxwell's Equation for Low-frequency Analysis

Equations for Electromagnetic Analysis

▪ Maxwell's Equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho \qquad \nabla \cdot \mathbf{B} = 0$$

▪ Constitutive Relation

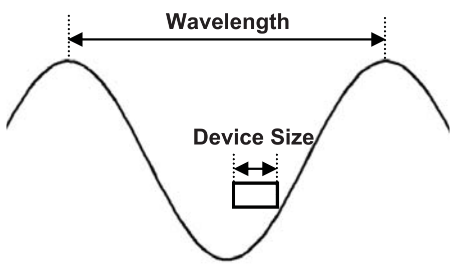
$$\mathbf{J} = \sigma \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H} \qquad \mathbf{D} = \varepsilon \mathbf{E}$$

▪ Continuity Equation

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

[Device Size << Wavelength] → Low-frequency Analysis



Rule:

Utilize Low-frequency Analysis when Device size < 0.1×Wavelength

* Wavelength of 1kHz=1000km

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Maxwell's Equation for Low-frequency Analysis

3 Types of Low-frequency Analysis

1. Electric and Magnetic Field Analysis (Voltage-fed analysis)

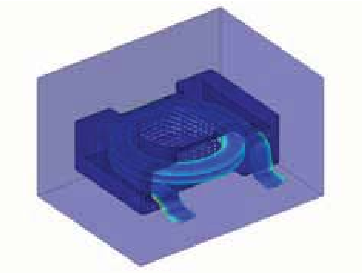
Solve (1) Electric scalar potential V and (2) Magnetic vector potential **A**

→ Find distribution of (1) current density **J** and (2) magnetic flux density **B**

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{B} = \mu \mathbf{H} \qquad \mathbf{J} = \sigma \mathbf{E}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

*** Generally 3D analysis**
*** Computationally expensive**



Power Inductor

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Maxwell's Equation for Low-frequency Analysis

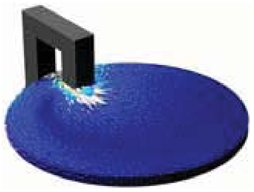
2. Eddy Current Analysis

Solve Magnetic vector potential **A**

→ Find (1) Eddy current density **J**,
(2) Magnetic flux density **B**

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \mathbf{J} = \mathbf{J}_e - \sigma \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \mu \mathbf{H} \qquad \mathbf{B} = \nabla \times \mathbf{A}$$



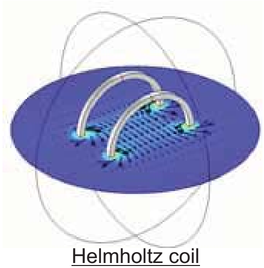
Eddy current in magnetic brake

3. Magnetostatic analysis

Solve Magnetic vector potential **A**

→ Find Magnetic flux density **B**

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \mathbf{B} = \mu \mathbf{H} \qquad \mathbf{B} = \nabla \times \mathbf{A}$$



Helmholtz coil

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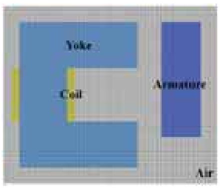
FEM Examples for Magnetostatic Analysis

- **Magnetostatic Analysis** of **2D/3D** magnetic actuator with/without **nonlinearity**

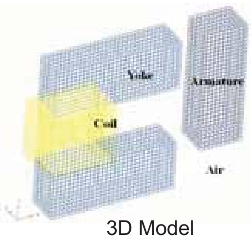
- **Result Comparison**

1) Developed Program 2) ANSYS v11 3) COMSOL v3.5

Magnetic Actuator



2D Model



3D Model

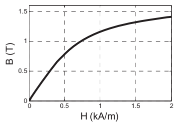
Linear problem

Constant μ of ferromagnetic material

Non-linear problem

$\mu = f(\mathrm{B})$

Saturation of ferromagnetic material



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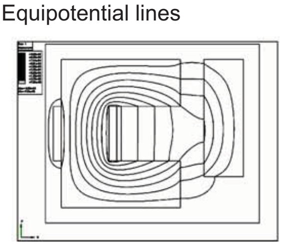
2D Linear Problem – Analysis Result

- **Linear Magnetostatic Equation** $\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{J}$ **(Constant μ)**

▪ Developed Program

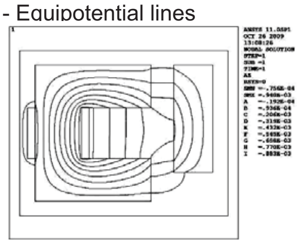
- Equipotential lines



- Magnetic energy : **14.832**

▪ ANSYS v11

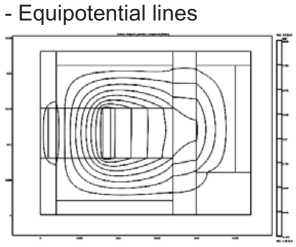
- Equipotential lines



- Magnetic energy : **14.832**

▪ COMSOL v3.5

- Equipotential lines



- Magnetic energy : **14.832**

* Quadrilateral nodal element – Vector potential **A**

* Exactly Identical Analysis results → No numerical issues

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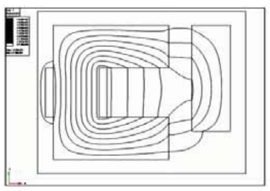
2D Non-linear Problem – Analysis Result

- **Non-linear Magnetostatic Equation** $\nabla \times \left(\frac{1}{\mu(\mathbf{A})} \nabla \times \mathbf{A} \right) = \mathbf{J}$

  * Newton-Raphson method $\mathbf{F}(\mathbf{A}) = \mathbf{K}(\mathbf{A})\mathbf{A} - \mathbf{J} = 0 \rightarrow \mathbf{A}^{(n+1)} = \mathbf{A}^{(n)} - \mathbf{F}(\mathbf{A}^{(n)}) / \mathbf{F}'(\mathbf{A}^{(n)})$ $\left(F'_{ij} = K_{ij} + \frac{\partial K_{ik}}{\partial A_j} A_k \right)$

▪ Developed Program

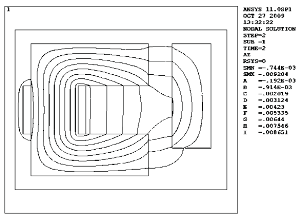
- Equipotential lines



- Magnetic energy : **14.325**

▪ ANSYS v11

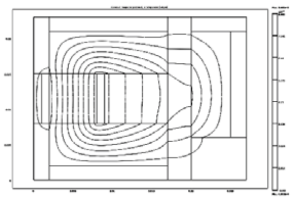
- Equipotential lines



- Magnetic energy : **14.325**

▪ COMSOL v3.5

- Equipotential lines



- Magnetic energy : **14.533**

* Identical unique solution → No numerical issues

* **COMSOL mistakenly use energy formulation of linear problem**

$$W_f = \int_V \left(\frac{1}{2} HB \right) dV \neq W_f = \int_V \left(\int_0^B H \, dB \right) dV$$

(**Linear**) (**Non-linear**)

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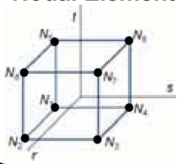
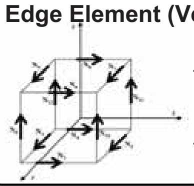
3D Problem – Edge Element

Nodal Element



- Three DOF (A_x, A_y, A_z) at each 8-node
- Scalar shape function

Edge Element (Vector Element)

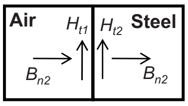


- One DOF (A_p) at each 12-edge
- Vector shape function

Edge element→ To satisfy boundary conditions at the interface.

- **Two Boundary Conditions at the Interface of Different Materials**

1. Continuous B_n ➔ $B_{n1}=B_{n2}$: Automatically satisfied
2. Continuous H_t ➔ $H_{t1}=H_{t2}$: B_t should jump at the interface $(\because (1/\mu_{air})B_{t1} = (1/\mu_{steel})B_{t2})$



$B_t = \frac{\partial A_n}{\partial t} - \frac{\partial A_t}{\partial n}$ → Nodal : Continuous
Edge : Allow discontinuity → ***Only edge element satisfies B.C. 2.***

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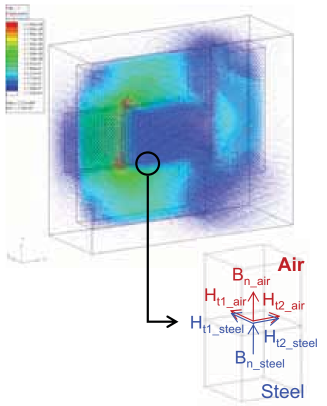
3D Problem – Edge Element

❑ **Example - Two Boundary Conditions in Nodal and Edge element**



	Nodal Element	Edge Element
B.C.1.	B_{n_air} = B_{n_steel} -0.20199 -0.20199	B_{n_air} = B_{n_steel} -0.19167 -0.19167
B.C.2.	H_{t1_air} ≠ H_{t1_steel} -568.950 -7368.444	H_{t1_air} ≈ H_{t1_steel} -506.553 -928.098
	H_{t2_air} ≠ H_{t2_steel} -6.349×10^{-9} 1.273×10^{-6}	H_{t2_air} ≈ H_{t2_steel} -2.067×10^{-3} 3.723×10^{-3}

Edge element gives accurate solution at the material interface

**** Magnetic field at the material interface is used for the magnetic force calculation***

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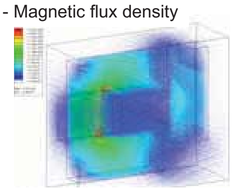
3D Linear Problem – Analysis Result

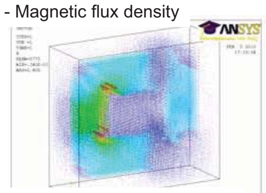
❑ **Edge Element**

▪ **Developed Program**
- Magnetic flux density



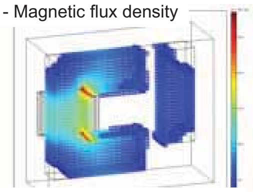
- Magnetic energy : **0.014924**

▪ **ANSYS v11**
- Magnetic flux density



- Magnetic energy : **0.014835**

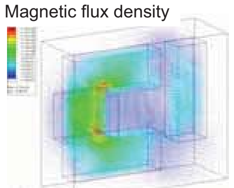
▪ **COMSOL v3.5**
- Magnetic flux density



- Magnetic energy : **0.014925**

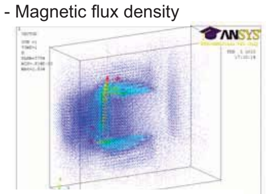
❑ **Nodal Element**

▪ **Developed Program**
- Magnetic flux density



- Magnetic energy : **0.015339**

▪ **ANSYS v11**
- Magnetic flux density



- Magnetic energy : **0.005303**

* Almost identical magnetic flux in edge element result

* Nodal element without gauge condition gives reasonable result

* **ANSYS nodal element fail (Due to gauge condition)**

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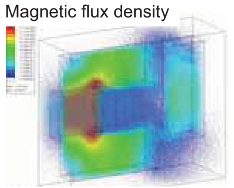
3D Non-linear Problem – Analysis Result

❑ **Edge Element**

▪ **Developed Program**

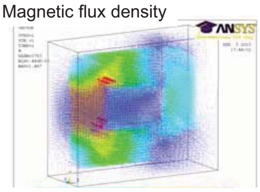
- Magnetic flux density



- Magnetic energy : **0.014733**

▪ **ANSYS v11**

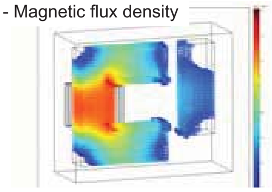
- Magnetic flux density



- Magnetic energy : **0.014644**

▪ **COMSOL v3.5**

- Magnetic flux density

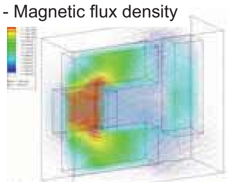


- Magnetic energy : **0.014801**

❑ **Nodal Element**

▪ **Developed Program**

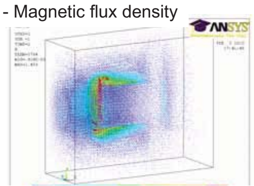
- Magnetic flux density



- Magnetic energy : **0.014475**

▪ **ANSYS v11**

- Magnetic flux density



- Magnetic energy : **0.005260**

* Almost identical magnetic flux in edge element result

* Nodal element without gauge condition gives reasonable result

* **ANSYS nodal element fail (Due to gauge condition)**

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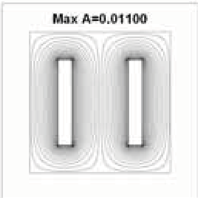
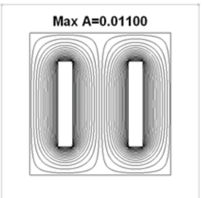
2D Eddy Current Analysis

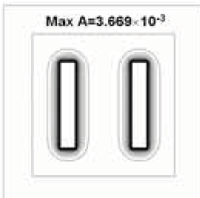
❑ **Time-harmonic linear equation:** $\nabla \times (\nabla \times \mathbf{A}) + j\omega\sigma\mathbf{A} = \mathbf{J}$

▪ **Developed Program**



▪ **COMSOL v4.2**





10 kHz | 50 kHz | 100 kHz

* Identical Analysis results → No numerical issues

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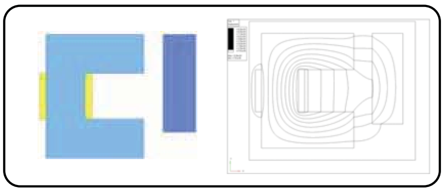
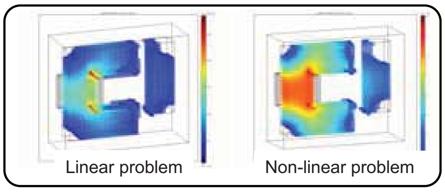
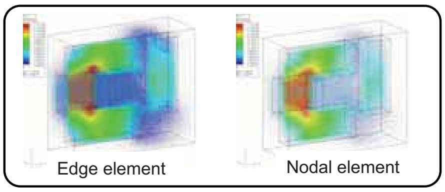
Summary – Low-frequency EM Field Analysis

- 2D/3D magnetostatic analysis in linear/non-linear problem is investigated
- No numerical issue is found in 2D analysis
- In 3D analysis, edge elements gives more accurate solution at the material interface, which is necessary for accurate magnetic force calculation



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(1) Low-frequency Electromagnetic Field Analysis

(2) Magnetic Force/Torque Calculation

(3) Structural Topology Optimization
1. Magnetic Actuator
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(4) Conclusion

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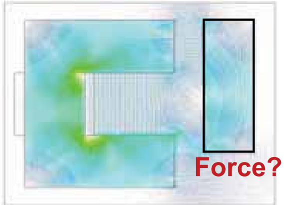
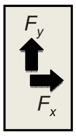
Magnetic Force Calculation

❑ Two Types of Magnetic Forces



Magnetic Field Distribution (FEM)

1. Total Force

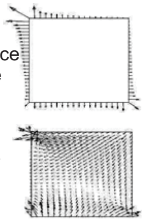


* *Rigid-body analysis* (single net force)

a. Global virtual work method

b. Sum of local (distributed) force

2. Distributed Force



* *Structural/Vibration analysis* (Force distribution)

a. Surface-force method

(1) Maxwell stress tensor (2) Equivalent current/charge (3) Local virtual work

b. Body-force method

(1) Maxwell stress tensor (2) Equivalent current/charge

**** Debate about true distributed force is still in progress***

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Distributed Force - Surface-force Method

❑ Formulation

❑ Maxwell stress tensor method

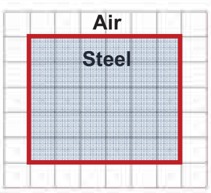
$$\mathbf{F}_s = \left[\frac{1}{\mu_0} B_n B_t\right]\mathbf{n} + \left[\frac{1}{2\mu_0}(B_n^2 - B_t^2)\right]\mathbf{t}$$

❑ Equivalent current method

$$\mathbf{F}_s = \left[\frac{\mu_0}{2}\left(\mu_r^2 - 1\right)H_t^2\right]\mathbf{n} + \left[\left(1-\mu_r\right)B_n H_t\right]\mathbf{t}$$

❑ Equivalent charge method

$$\mathbf{F}_s = \left[\frac{1}{2\mu_0}\left(1-\mu_r^2\right)B_n^2\right]\mathbf{n} + \left[\left(1-\frac{1}{\mu_r}\right)B_n H_t\right]\mathbf{t}$$



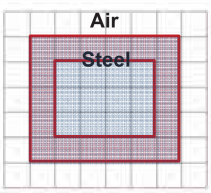
* Utilize Magnetic field at the material interface

❑ Local virtual work method

$$\mathbf{F}_s = \int\left[\left\{-\frac{1}{\mu}\mathbf{B}^T\mathbf{J}^{-1}\frac{\partial \mathbf{J}}{\partial x_i}\mathbf{B}\right\} + \left\{\left(\int_0^B \frac{1}{\mu}\tilde{B}\,d\tilde{B}\right)\left|\mathbf{J}\right|^{-1}\frac{\partial\left|\mathbf{J}\right|}{\partial x_i}\right\}\right]dV$$

* Most accurate method (suitable with FEM)

* Complicated calculation process if **B-H** relation is non-linear



* Utilize Magnetic field at the boundary elements

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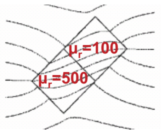
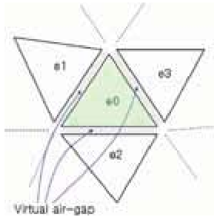
Distributed Force - Body-force Method

- Virtual air gap scheme to calculate force of different materials in contact
- Extended for body force calculation method



- Procedure
  - * Split object into small bodies (finite elements)
  - * Calculate Magnetic field $\mathbf{B}_a$ and $\mathbf{H}_a$ at virtual air gap between finite elements
  - * Calculate magnetic force applied to each finite element using conventional method
  - * Dividing the magnetic force by volume



$$\mathbf{B}_a = B_{n1}\mathbf{n} + \frac{B_{t1}}{\mu_{r1}}\mathbf{t} = B_{n2}\mathbf{n} + \frac{B_{t2}}{\mu_{r2}}\mathbf{t}$$

$$\mathbf{H}_a = \mu_{r1}H_{n1}\mathbf{n} + H_{t1}\mathbf{t} = \mu_{r2}H_{n2}\mathbf{n} + H_{t2}\mathbf{t}$$

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Magnetic Field Correction for 2D Analysis

- **Correct magnetic field at the material interface**

  * Necessary in 2D force calculation using magnetic field at the material interface.

  Local virtual work method does not need the correction

  **** How to correct magnetic field***

  → In a way to satisfy two boundary conditions

Two boundary conditions at the material interface

1. Continuous B_n

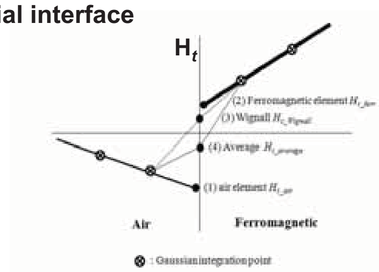
→ Automatically satisfied

2. Continuous H_t

→ Correction

$$\textbf{Wignall}(1988):\ H_{t\_w} = \frac{H_{t\_air}\mu_0 + H_{t\_ferr}\mu_f}{\mu_0 + \mu_f}$$

$$\textbf{Averaging}:\ H_{t\_av} = \frac{H_{t\_air} + H_{t\_ferr}}{2}$$



Correction of H_t at the interface

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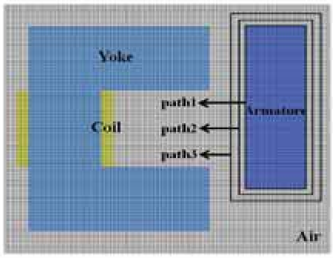
Numerical Example

❑ **2D Magnetic Actuator**

* Calculation of <u>Total / distributed force</u>
* Wignall / Averaging method to correct H_t

<u>2D magnetic actuator</u>



Three integration paths of Maxwell stress tensor method

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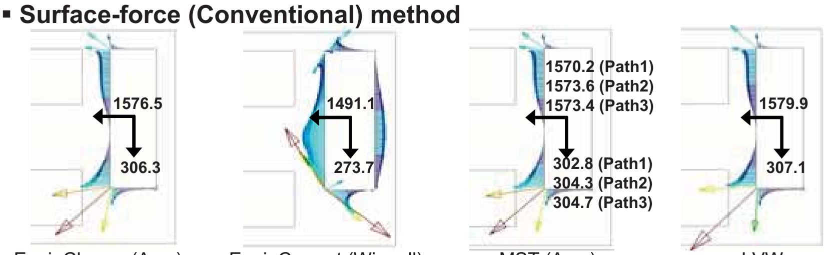
Magnetic Force Calculation

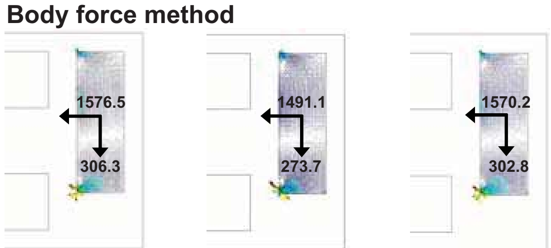
❑ **Result Comparison: Total (Global) Magnetic Force**

▪ **Surface-force (Conventional) method**



Equi. Charge (Avg.) Equi. Current (Wignall) MST (Avg.) LVW

▪ **Body force method**



Equi. Charge Equi. Current MST

* **Total force**: Almost <u>identical</u> in <u>every method</u> with <u>magnetic field correction</u>

* **Distributed Force**
  - Different distribution in surface and body force

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Summary – Magnetic Force Calculation

- There are two types of magnetic force. Total force is a single net force acting on rigid body. Distributed force is necessary for structural/vibration analysis.
- Every force calculation method gives same total force with the magnetic field correction at the material interfaces.
- Different force calculation method gives different distributed force. Further investigation is needed to decide true distributed force

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(1) Low-frequency Electromagnetic Field Analysis

(2) Magnetic Force/Torque Calculation

(3) Structural Topology Optimization

1. Magnetic Actuator
2. Inductor
3. Switched Reluctance Motor

(4) Conclusion

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Structural Topology Optimization

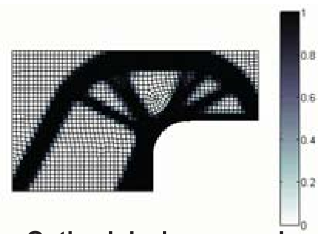
- **Structural Topology Optimization:**
  Mathematical approach to find optimal geometry (size/shape/number of holes)

  - **Mathematical Representation of Geometry using Material Density ρ**

Density ρ of each finite element
ρ=0 → Material 1 (air)
ρ=1 → Material 2 (iron)

+

Material properties: function of density ρ
ρ=0 → μ_r=1 (air)
ρ=1 → μ_r=2000 (Iron)



Optimal design example
* White → ρ=0 (air)
* Black → ρ=1 (steel)

Optimal material density ρ distribution of finite elements → Optimal geometry

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Application to Low-frequency EM Problem

1. Magnetic Actuator
- **(A-1) Maximize total force – Electromagnet**
- **(A-2) Maximize total force – Permanent magnet**
- **(B) Maximize total force + Maximize stiffness**
- **(C) Ferromagnetic/Coil/Magnet material design**

2. Inductor
- **(A) Coil/Core design at DC**
- **(B) Coil design at AC**

3. Switched Reluctance Motor
- *** 2D/3D design**

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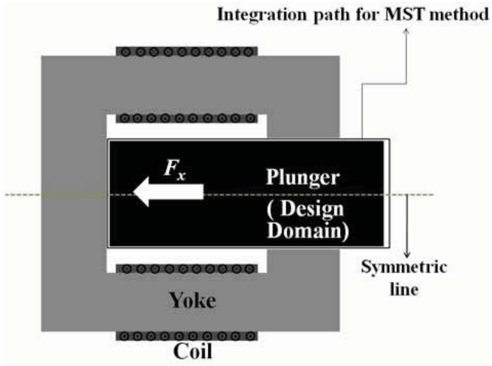
1. Magnetic Actuator – (A-1) Maximize Total Force

❑ Optimization problem – Electromagnet

Find	**Optimal <u>ferromagnetic</u> material distribution of <u>plunger</u>**
Maximize	**<u>Total force F_x</u> acting on plunger**
Subject to	**Constraint on <u>plunger volume V</u>**



<u>Actuator operated by electromagnet</u>

- **Magnetic Force Calculation**

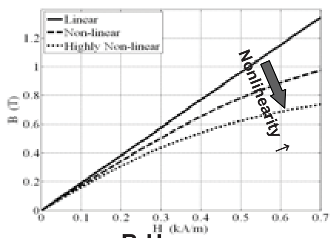
  1) Maxwell stress tensor method 2) Local virtual work method

1. Magnetic Actuator – (A-1) Maximize Total Force

❑ Optimization Result – Saturation effect of ferromagnetic material

Linear (Constant μ) Non-linear ($\mu=f(\mathbf{B})$) Highly non-linear ($\mu=f(\mathbf{B})$)

Higher nonlinearity



B-H curve

<u>Comment</u>

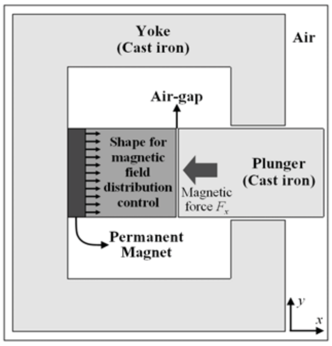
* **<u>Optimal shape</u>** minimize **<u>magnetic reluctance</u>** to **<u>increase magnetic flux</u>** at the air-gap

* **<u>Saturation</u>** lead to **<u>thick optimal shape</u>**

1. Magnetic Actuator – (A-2) Maximize Total Force

❑ Optimization problem – Permanent Magnet

Find	**Optimal ferromagnetic material distribution at yoke near air-gap**
Maximize	**Total force F_x acting on plunger**
Subject to	**Constraint on volume V**



Actuator operated by permanent magnet

* **Permanent magnet** may produce **same air-gap magnetic flux**

* Design domain controls **distribution** of **air-gap magnetic field**

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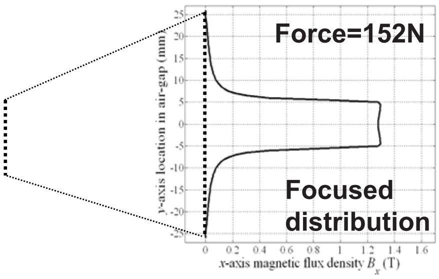
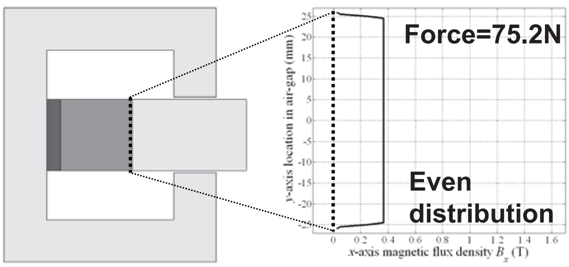
1. Magnetic Actuator – (A-2) Maximize Total Force

❑ Optimization Result



❑ Rectangular shape for comparison



Comment

* **Optimal structure** produce **twice larger force** than **rectangular shape**.

* **Force increase** is due to **focused distribution** at the **same magnetic flux** by **permanent magnet**

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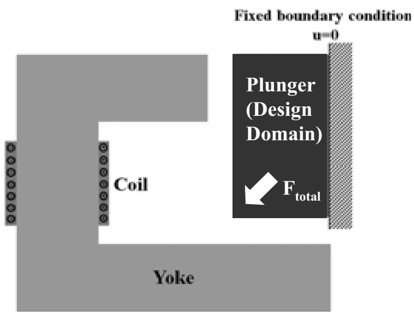
1. Magnetic Actuator – (B) Maximize Stiffness

❑ Optimization problem

Find	**Optimal <u>ferromagnetic</u> material distribution of <u>plunger</u>**
Maximize	**1) <u>Total force</u>** F_x
	2) <u>Stiffness</u>
Subject to	**60% plunger volume**



- **Magneto-Structural Analysis**

Magnetostatic analysis → (Magnetic field **B**) → **Distributed Magnetic force** → (Force distribution) → **Structural analysis**

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1. Magnetic Actuator – (B) Maximize Stiffness

❑ Optimization Result I – Surface force (Local virtual work method)

Magnetic field   Distributed force   Deformed shape

Comment

* **<u>Upper right structure</u>** is for **<u>stiffness</u>**
<u>Other structure</u> is for **<u>magnetic force</u>**

* **<u>Two different force distribution</u>** gives totally **<u>different optimal structure</u>**

❑ Optimization Result II – Body force

Magnetic field   Distributed force   Deformed shape

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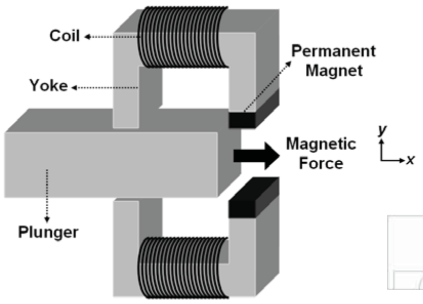
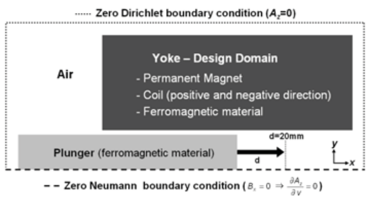
1. Magnetic Actuator – (C) Coil / Magnet Design

❑ **Optimization problem**

Find	**Optimal material distribution of <u>ferromagnetic /coil / magnet</u> material <u>at the same time</u>**
Maximize	**<u>Average force</u> on plunger (20mm movement)**
Subject to	**Constraint on <u>volume of each component</u>**





d=0mm d=10mm d=20mm

Plunger movement

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1. Magnetic Actuator – (C) Coil / Magnet Design

❑ **Optimization Result – Magnet Magnetization direction**

45˚ 90˚

135˚ 180˚

<u>Comment</u>

* **<u>Coil</u>** near the **<u>air-gap</u>** to **<u>minimize leakage</u>**

* **<u>Magnetization direction</u>** is aligned with magnetic field of **<u>ferromagnetic material</u>** generated by **<u>coil</u>**

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1. Magnetic Actuator – (C) Coil / Magnet Design

- Optimization Result – External current strength

1Amp 3Amp

5Amp 7Amp

Comment

* **Weak** external current
→ Magnet **inside** ferromagnetic material

* **Strong** external current
→ Magnet **outside** ferromagnetic material

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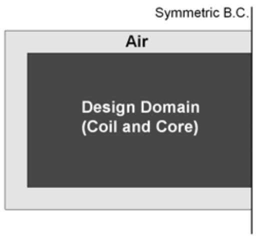
2. Inductor – (A) Coil and Core Design at DC

- **Optimization problem**

Find	**Optimal material distribution of inductor coil / core at the same time**
Maximize	**Inductance *L***
Subject to	**Constraint on volume *V* of each component**



- **Optimization result**

***V*=40% *L*=14.5μH**

***V*=60% L=27.4μH**

***V*=80% *L*=36.0μH**

***V*=100% L=36.3μH**

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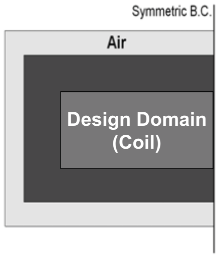
2. Inductor – (A) Coil Design at AC

❑ Optimization problem

Find	**Optimal copper material distribution of inductor coil**
Minimize	**Inductance *L***
Subject to	**Constraint on volume of coil**



❑ Optimization result

Optimal AC
Coil shape

Current density
distribution

1kHz AC **5kHz AC** **10kHz AC**

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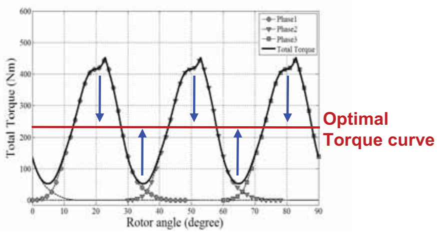
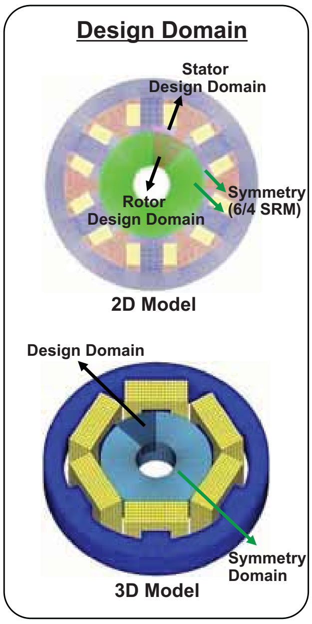
3. Switched reluctance motors

❑ Optimization Problem Definition

Find	**(1) Optimal ferromagnetic material distribution of stator/rotor** **(2) Optimal voltage turn-on/off angles**
Minimize	**(1) Torque ripple with constant average torque** $\sum_i \left(T_i / T_{average} - 1.0\right)^2$ **(2) Mass**
Subject to	**Constraint on copper loss (=$i_{rms}^2 R$)**

Torque Curve
(High torque ripple)





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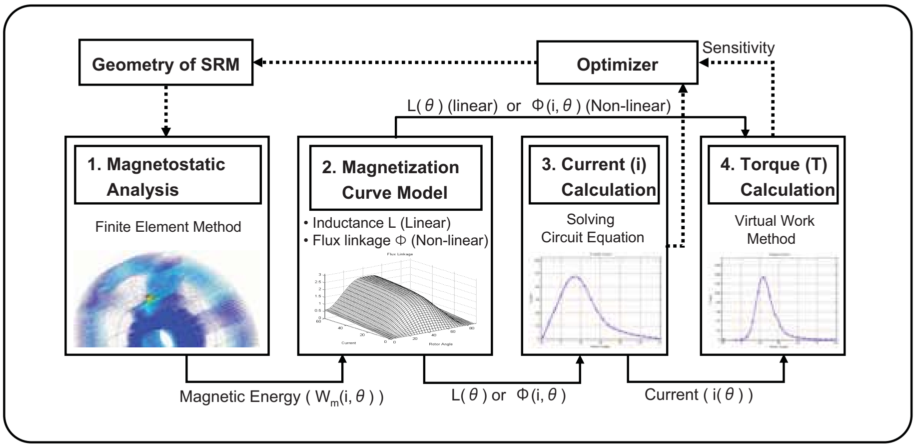
3. Switched reluctance motors

❑ Performance Analysis: Steady-state Operation

* **Analytical representation** of **current** and **torque** for sensitivity analysis

* Design with **Linear (Constant μ)** and **non-linear (μ=f(B)) ferromagnetic material**



Flowchart of performance analysis

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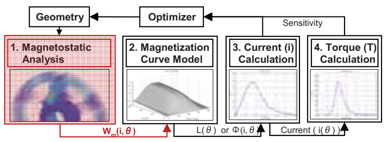
3. Switched reluctance motors

❑ Performance analysis

- **Magnetostatic analysis**



	Linear (Constant μ)	Non-linear (μ=f(B))
Magnetostatic equation	$\nabla\times\left(\frac{1}{\mu}\nabla\mathbf{A}\right)=\mathbf{J}\;\Rightarrow\;\mathbf{KA}=\mathbf{J}$	$\nabla\times\left(\frac{1}{\mu(B^2)}\nabla\mathbf{A}\right)=\mathbf{J}\;\Rightarrow\;\mathbf{K}(\mathbf{A})\mathbf{A}=\mathbf{J}$
Solve Equation	$\mathbf{A}=\mathbf{K}^{-1}\mathbf{J}$	*Newton-Raphson Iteration $\mathbf{A}^{(n+1)}=\mathbf{A}^{(n)}-\mathbf{F}(\mathbf{A}^{(n)})/\mathbf{F}'(\mathbf{A}^{(n)})$
Calculate magnetic energy	$W_m=\int_{\text{allspace}}\frac{1}{2\mu}B^2\,dv$ * $\mathbf{B}=\nabla\times\mathbf{A}$	$W_m=\int_{\text{allspace}}\left(\int_0^B\frac{1}{\mu(B^2)}B\,dB\right)dv$ * $\mathbf{B}=\nabla\times\mathbf{A}$

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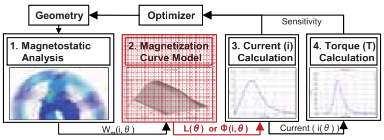
3. Switched reluctance motors

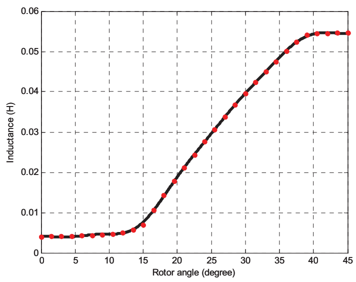
- Performance analysis
  - Magnetization curve modeling



Linear : Magnetic energy $W_m = \frac{1}{2} L i^2$ Inductance L

Non-linear : Magnetic energy $W_m = \int i \, d\Phi$ Flux linkage Φ

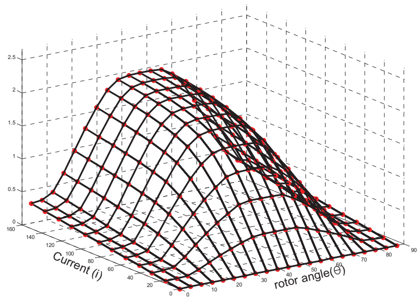
Linear – Inductance L



Analytical Expression of Inductance L

$$L(\theta) = a_0 + \sum_k a_k \cos N_r n\theta$$

Non-linear – Flux Linkage Φ



Analytical Expression of Flux Linkage Φ

$$\Phi_m(i,\theta) = \left(F_{1m,0} i^2 + F_{2m,0} i + F_{3m,0}\right) + \sum_{n=1}^{NF} \left(F_{1m,n} i^2 + F_{2m,n} i + F_{3m,n}\right) \cos(nP_r\,\theta)$$

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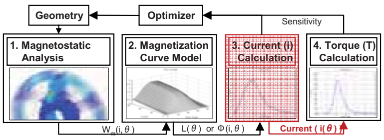
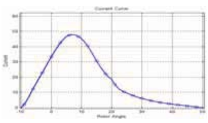
3. Switched reluctance motors

- Performance analysis
  - Current Calculation





Calculate current curve $i(\theta)$ by solving voltage equation

Linear

Analytical Expression of Inductance L

$$L(\theta) = a_0 + \sum_k a_k \cos N_r n\theta$$

Non-linear

Analytical Expression of Flux Linkage Φ

$$\Phi_m(i,\theta) = \left(F_{1m,0} i^2 + F_{2m,0} i + F_{3m,0}\right) + \sum_{n=1}^{NF} \left(F_{1m,n} i^2 + F_{2m,n} i + F_{3m,n}\right) \cos(nP_r\,\theta)$$

Solve voltage equation $V = \cancel{Ri} + \frac{d\Phi(i,\theta)\;(= L(\theta) i)}{dt}$

Differentiable function for Current $i(\theta)$

$$i(\theta) = \frac{V_0\theta}{\omega L(\theta)} = \frac{V_0\theta}{\omega\left(a_0 + \sum_k a_k \cos N_r n\theta\right)}$$

Differentiable function for Current $i(\theta)$

$$i_p(\theta) = \frac{-X_2 \pm \sqrt{X_2^2 - 4X_1\left(X_3 - \Phi_p - \frac{V_0}{\omega}(\theta - \theta_p)\right)}}{2X_1} \qquad \left(X_i = \left(\sum_{n=0}^{NF} F_{im(p),n} \cos(nP_r\,\theta)\right)\right)$$

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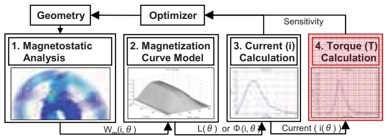
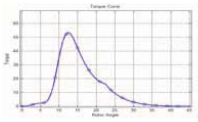
3. Switched reluctance motors

❑ Performance analysis

- **Torque Calculation**





Calculate torque curve $T(\theta)$ by using global virtual work method

Linear

Inductance $L(\theta)=a_0+\sum_k a_k\cos N_r n\theta$

Current $i(\theta)=\frac{V_0\theta}{\omega L(\theta)}=\frac{V_0\theta}{\omega\left(a_0+\sum_k a_k\cos N_r n\theta\right)}$

Non-linear

Flux Linkage $\Phi_m(i,\theta)=\left(F_{1m,0}i^2+F_{2m,0}i+F_{3m,0}\right)+\sum_{n=1}^{NF}\left(F_{1m,n}i^2+F_{2m,n}i+F_{3m,n}\right)\cos(nP_r\theta)$

Current $i_p(\theta)=\frac{-X_2\pm\sqrt{X_2^2-4X_1\left(X_3-\Phi_p-\frac{V_0}{\omega}(\theta-\theta_p)\right)}}{2X_1}$ $\left(X_i=\left(\sum_{n=0}^{NF}F_{im(p),n}\cos(nP_r\theta)\right)\right)$

Virtual work method $T(\theta,i)=\frac{\partial W_{co}(\theta,i)}{\partial\theta}\Big|_{i=\text{constant}}\left(W_{co}(\theta,i)=\int_0^i\Phi(\theta,i)di\Big|_{\theta=\text{constant}}\right)$

Differentiable function for torque $T(\theta)$ (linear)

$$T(\theta)=\frac{(V_0\theta)^2}{2\omega^2}\left(-\sum_{n=1}^{M}nN_ra_n\sin(N_rn\theta)\right)/\left(a_0+\sum_{n=1}^{M}a_n\cos(N_rn\theta)\right)^2$$

Differentiable function for torque $T(\theta)$ (non-linear)

$$T(\theta,i)=-\sum_{n=0}^{NF}\left(\frac{1}{3}F_{1m,n}\left(i^3-i_{2m-1}^3\right)+\frac{1}{2}F_{2m,n}\left(i^2-i_{2m-1}^2\right)+F_{3m,n}\left(i-i_{2m-1}\right)\right)nP_r\sin(nP_r\theta)$$
$$-\sum_{k=1}^{m-1}\sum_{n=0}^{NF}\left(\frac{1}{3}F_{1k,n}\left(i_{2k+1}^3-i_{2k-1}^3\right)+\frac{1}{2}F_{2k,n}\left(i_{2k+1}^2-i_{2k-1}^2\right)+F_{3k,n}\left(i_{2k+1}-i_{2k-1}\right)\right)nP_r\sin(nP_r\theta)$$

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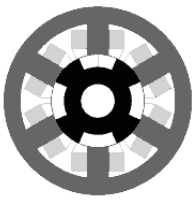
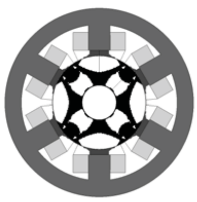
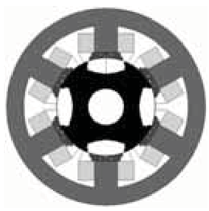
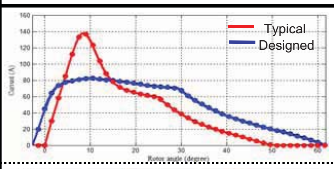
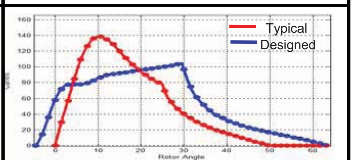
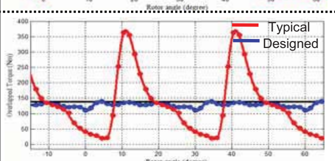
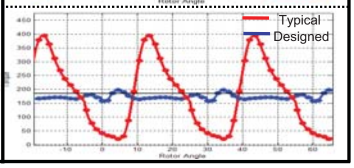
3. Switched reluctance motors

❑ Optimization Result – 2D Linear / Non-linear

Typical shape	Linear (Constant μ) Result	Non-linear ($\mu=f(B)$) Result
	 θ_{on}: -2.8° θ_{off}: 61.7°	 θ_{on}: -4.4° θ_{off}: 63.5°
Current (i) Curve		
Torque (T) Curve		

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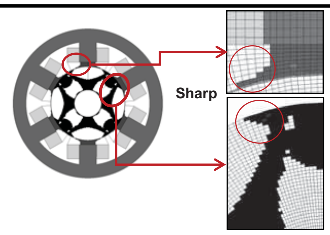
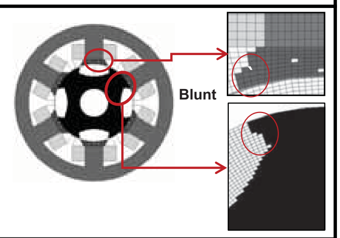
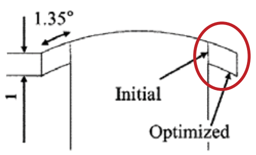
3. Switched reluctance motors

❑ Optimization Result Comparison – 2D Linear / Non-linear

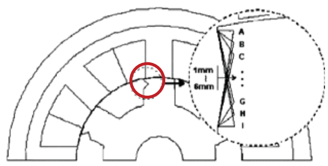
Typical shape	Linear Result	Non-linear Result
Magnetic Saturation	X	O
Volume	46.4% (Holes)	70.7% (No holes)







(pole shoe - Choi et. al. 2007)


(Notched teeth - Lee et. al., 2004)

Notched Shape

- Good Agreement with Previous research

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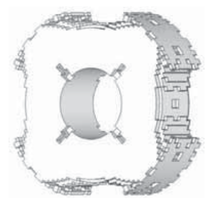
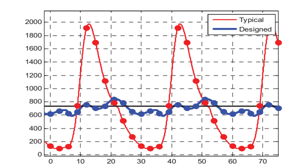
3. Switched reluctance motors

❑ Optimization Result – 3D Linear Edge/Nodal

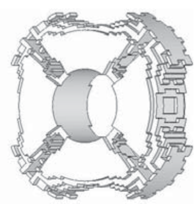
3D Edge Element

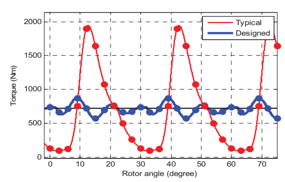




Torque (*T*) Curve

3D Nodal Element





Torque (*T*) Curve

**** 3D Notched Shape***

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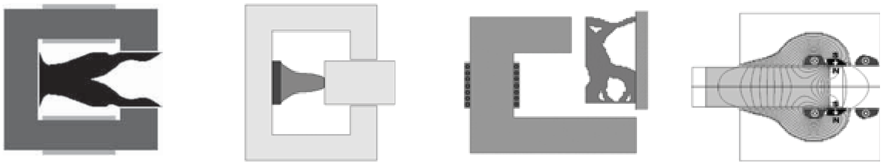
Summary – Structural Topology Optimization

1. Magnetic Actuator – Maximize Magnetic Force



2. Inductor – Maximize Inductance

3. Switched Reluctance Motor – Minimize Torque ripple

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(1) Low-frequency Electromagnetic Field Analysis

(2) Magnetic Force/Torque Calculation

(3) Structural Topology Optimization

1. Magnetic Actuator
2. Inductor
3. Switched Reluctance Motor

(4) Conclusion

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4. Conclusions

- **Low-frequency Electromagnetic Analysis**
  - There are three analysis types for low-frequency analysis
  - No numerical issues in 2D analysis using finite element method
  - 3D edge element gives accurate solution at the material interface ,which is needed for accurate force calculation

- **Magnetic Force/Torque Calculation**
  - In 2D analysis, the magnetic field correction at the material interface enables us to obtain same total force in every force calculation method
  - Debate about true distributed force is still in progress

- **Structural Topology Optimization**
  - The design examples of magnetic actuator, inductor, switched reluctance motors are presented. The optimal shapes successfully satisfy each specific design goal.

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