

Applications of modeFRONTIER in Stochastic Model Extrapolation and Robustness Design

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INTRODUCTION

In vehicle design, response surface model (RSM) is commonly used as a surrogate of the high fidelity Finite Element (FE) model to reduce the computational time and improve the efficiency of design process. However, RSM introduces additional sources of uncertainty, such as model bias, which largely affect the reliability and robustness of the prediction results. The bias of RSM need to be addressed before the model is ready for extrapolation and design optimization. This paper employed the commercial available optimization platform modeFRONTIER to investigate the Bayesian inference based model extrapolation method which is previously proposed by the authors, and provides a systematic and integrated stochastic bias corrected model extrapolation and robustness design process under uncertainty. A real world vehicle design example is used to demonstrate the validity of the proposed method.

KEYWORDS

modeFRONTIER, Vehicle Design, Model Validation, Model Extrapolation, Robust Design

INTRODUCTION

Computational Aided Engineering (CAE) becomes a vital tool in various industries. In automotive industry, Finite Element (FE) models are widely used in vehicle design. Model validation [1-6] is the process of comparing CAE model outputs with test measurements in order to assess the validity and predictive capabilities of the CAE model for its intended usage. Successful implementation of model validation will increase the chance of virtual testing and may lead to significant reduction in prototype building and testing of vehicle designs.

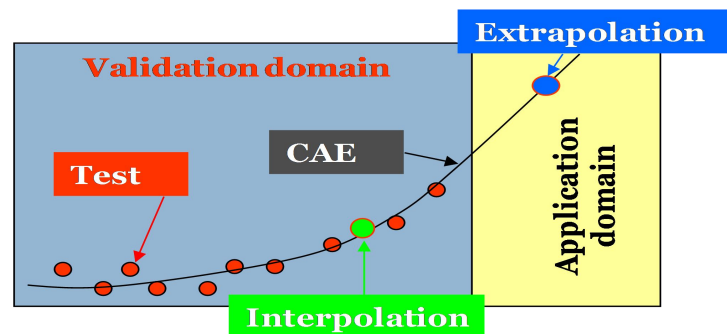
Various statistical inference techniques have been developed [12]. However, these methods did not fully address the needs for design interpolation and extrapolation under uncertainty. With these considerations, three interpolation and extrapolation methods are investigated by Zhan et al.[20]. They are Bayesian inference-based method [21], Gaussian process modeling (GPM)-based method [22], and Copula-based method [23]. Three stochastic validation metrics, such as area metric, reliability-based metric, and Bayesian confidence metric, are used to evaluate the predictive capability of these three methods.. It is found that all three methods are capable of providing good predictions for the purpose of interpolation and extrapolation. Among the three methods, the Bayesian

inference-based method is more straightforward, effective, has less tuning parameters, and is easy for engineering implementation. The Gaussian process method shows superiority in capturing the mean of the responses when less training data are available. However, it might be difficult to determine the hyper-parameters of the Gaussian process when solving high-dimensional problems. In addition, the drastically changing prediction intervals might have some potential negative effects on identification of a better design. Copula-based method is able to capture general statistical relationships between model prediction and model bias, and between design variables and model bias. This method may be good to handle large bias noisy situation. However, the implementation process is complex.

Reliability-based design optimization (RBDO) is a frequently used approach in design under uncertainty aiming at obtaining a design solution that is most reliable under the variations of design variables. Beyond deterministic design optimization, which often drives the optimal design to the limit of design constraints and leaves little or no latitude for uncertainty, RBDO considers the uncertainty of controllable or uncontrollable variables associated with an existing CAE model. However, most of the traditional RBDO formulations do not account for the uncertainty from the CAE model; a CAE model is believed to be accurate enough for reliability assessment before it is implemented in RBDO process. RBDO fails if the model is not accurate and is affected with large model uncertainty. Therefore, model bias correction is critical before implementing RBDO.

The rest of the paper is organized as follows: The Bayesian inference based stochastic interpolation and extrapolation method is briefly introduced, Next, RBDO formulation is introduced and a vehicle frontal impact case study are used to investigate the capability of the proposed approach, and the FE simulations of the robust design are used to confirm the results. Finally, some discussions and summary are given in the end.

STOCHASTIC INTERPOLATION & EXTRAPOLATION METHODS



Model interpolation and extrapolation

In numerical analysis, interpolation is a method of constructing new data points within the range of a discrete set of known data points. Extrapolation is a process in which information is gained by obtaining function from the known data, and then estimating the value of a new data point which is beyond the range of given data by using the extended (extrapolated) function as the source. Typically, the quality of a particular extrapolation method is limited by the assumptions about the function made by the

method. For example, if a method assumes the data are smooth, then a non-smooth function will be poorly extrapolated. Even for proper assumptions about the function, the extrapolation can diverge strongly from the function. In general, CAE models are not developed solely for single design point analysis. It is required that the CAE model possesses dependable predictive capability under different design scenarios within a predefined design space. Additionally, CAE models can have a large amount of inputs. It is desirable that a CAE model is valid in a wide range of design variables (for design space exploration). A CAE model may have to be validated for the entire range of its inputs before it can be claimed as a "globally validated model", which will always yield designs that will perform as predicted. This requires a vast amount of resources to conduct multiple tests at each point of a sufficiently large data set in an extremely high-dimensional space defined by the model inputs. However, due to limited resources and time constraints, the model validation activities are usually conducted in a well-controlled environment where only a limited number of physical tests are available

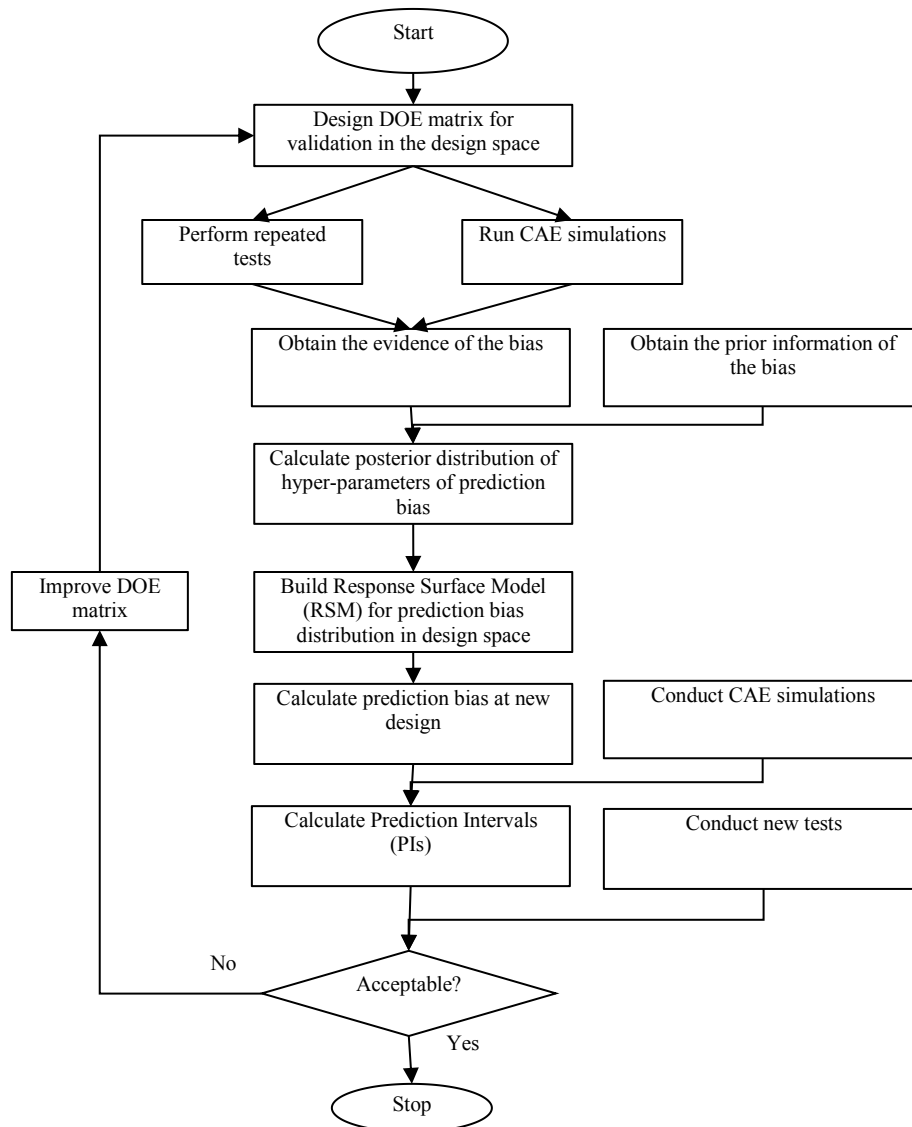
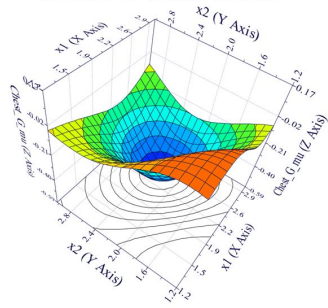


Figure 1 –Flowchart of Bayesian based Model extrapolation

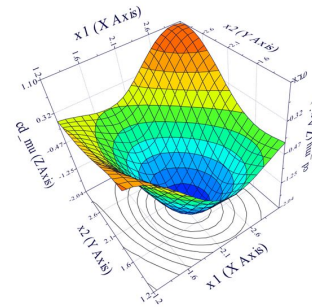
Zhan et al [21] proposed a Bayesian inference-based model interpolation and extrapolation method. This method utilizes the advantages of Bayesian inference and response surface model (RSM). The process starts with design of experiment (DOE) matrix for validation in the design space, followed by performing repeated physical tests and CAE simulations. The difference between test and CAE data are then calculated as evidence for the Bayesian updating of hyper-parameters of the bias distribution. After obtaining the prior information, the posterior distributions of prediction bias hyper-parameters are calculated.

Two RSMs are built for the mean and standard deviation of the prediction bias. After obtaining the CAE simulation result, the prediction intervals (PIs) of the output at new designs are calculated. Confirmation tests at new designs are used to validate the prediction results. If the test results of a new design are within the corresponding PIs, the prediction by interpolation or extrapolation is considered successful. Based on the result, the decision maker then decides to accept or reject the prediction result. If accepted, it is then passed to the downstream engineers for design optimization or robust design. On the other hand, if the prediction is rejected, additional DOE will be added and repeat the process until good quality prediction is achieved [21].

► Chest G bias mean $\tau_{\mu}(x_a)$



Crash Distance bias mean $\tau_{\mu}(x_a)$



RELIABILITY-BASED DESIGN OPTIMIZATION (RBDO)

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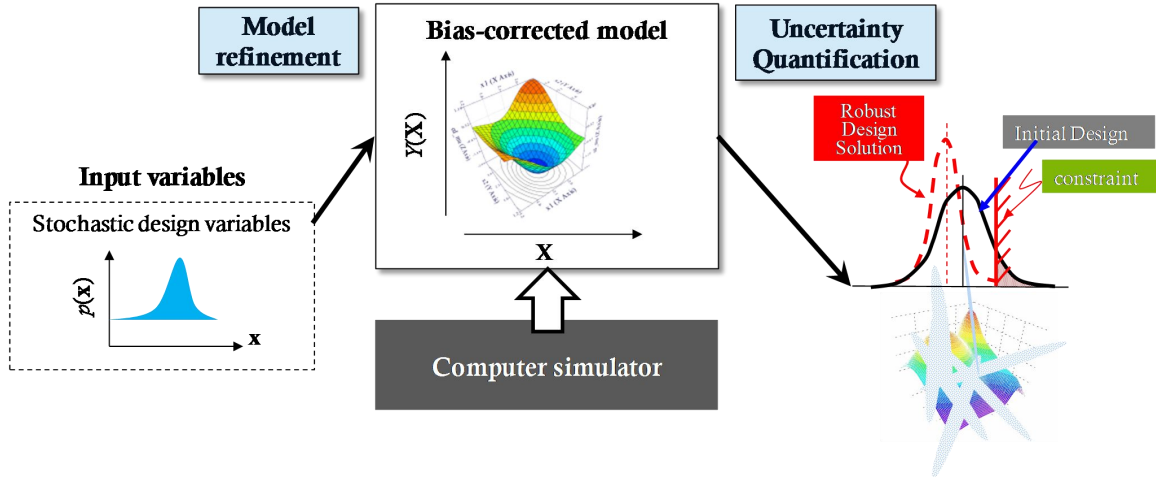


Figure 1 RBDO illustration

Minimize $E[f_0(X)]$
subject to $\Pr\{f_i(X) \leq 0\} \geq \alpha_i\%, i = 1, \dots, k$

X is a random vector that follows a known distribution, such as normal distribution, $E[\bullet]$ is the expectation of a function, $f_i(X), i = 1, \dots, k$ is a set of functional assessment of interest, and $\alpha_i\%, i = 1, \dots, k$, is the reliability level. Most of the existing RBDO works only consider the uncertainty associated with model input variables where $f_i(X)$ is deterministic by itself. On the other hand, if model uncertainty is considered and model bias correction is conducted beforehand, then $f_i(X), i = 1, \dots, k$, would be *stochastic* functions with which each single design X would provide a prediction distribution instead of a single prediction value. In this case, a modified approach is required for reliability assessment.

FRONTAL IMPACT CASE

Three stochastic interpolation and extrapolation methods are also applied to a vehicle design example. A Ford Taurus model from National Crash Analysis Center (NCAC) was used for this study [30]. Figure 2 shows the physical test and CAE model for the full frontal impact. The simulation speed is 56.6 km/h against a rigid wall. The comparison of test and CAE results show that CAE results are in a reasonable agreement with the physical test. Eight members of the front-end structure are chosen as the design variables, and two performance measures in a full frontal impact, chest G and crush distance are used as the performance measures. The detailed description of this case can be referred to Shi et al [31] and Zhan et al [22].



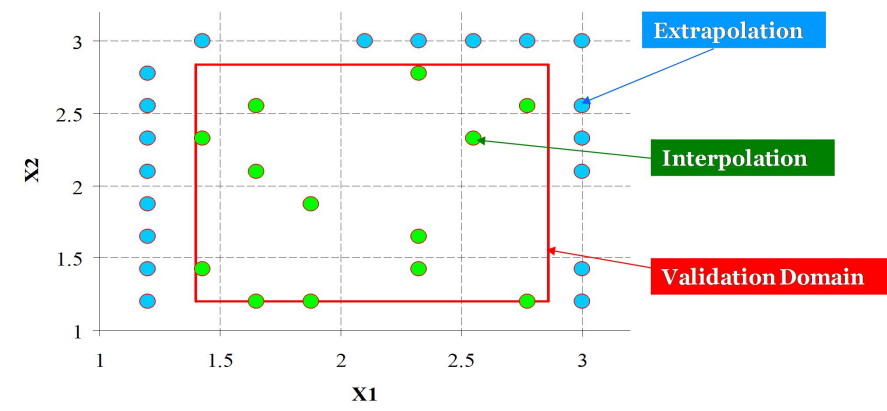
(a)

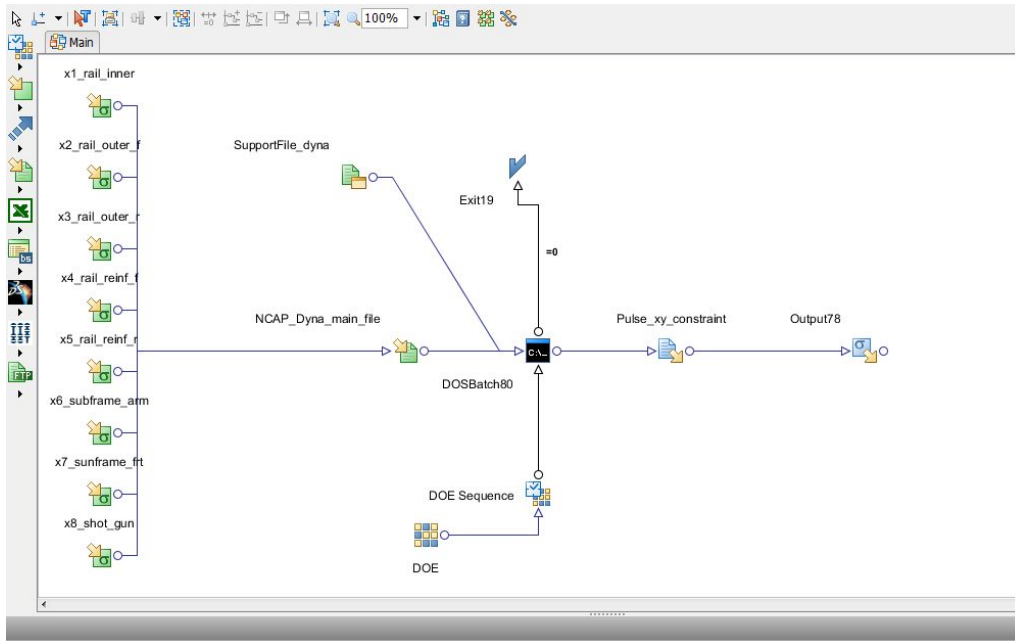


(b)

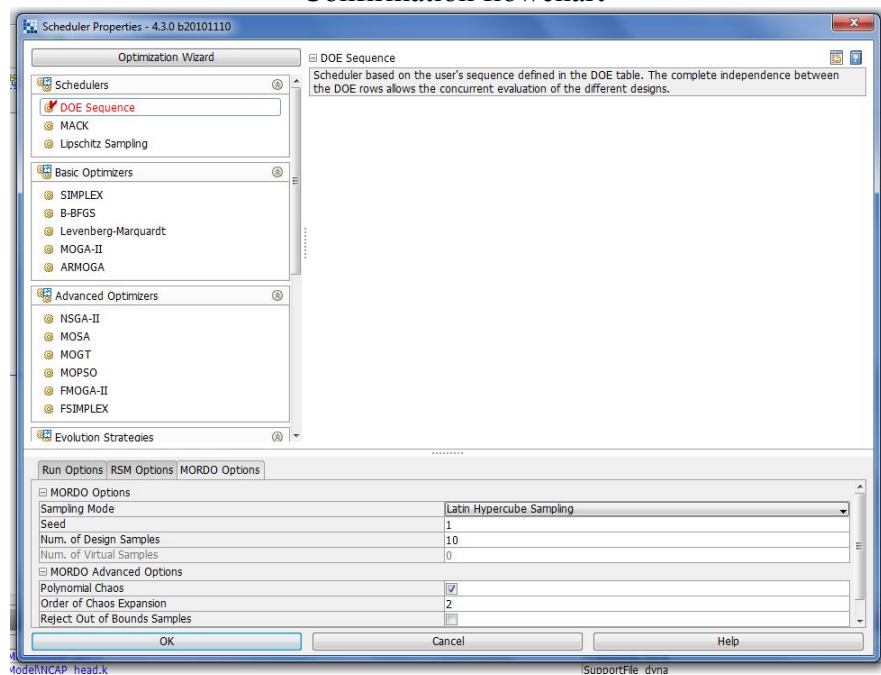
Figure 2 Comparison of structure deformation between: (a) test, (b) CAE model

In this benchmark problem, 80 uniform DOEs are generated in the design space (validation domain). Three repeated tests and a CAE simulation are conducted at each design configuration. 65 out of the original 80 DOE samples are used to construct the RSMs. The remaining 15 out of original 80 DOE samples are used as interpolation validation designs to investigate three stochastic interpolation and extrapolation methods. As shown in Zhan [21], the Bayesian inference-based method is more straightforward, effective, has less tuning parameters, and is easy for engineering implementation.\





Confirmation flowchart



RBDO setting

Figure shows the FE simulation results for the 10 designs generated from the traditional RBDO solution. Note that the chest G and crush distance results have been normalized. Apparently this is a failed solution; 1 out of 10 designs fails the Chest G constraint while all 10 designs fail the Crush Distance constraint. The probability distributions of Chest G and Crush Distance given by MCS are way off the distributions from the 10 FE simulations, and the reliabilities of the two constraints are neither close to 99%.

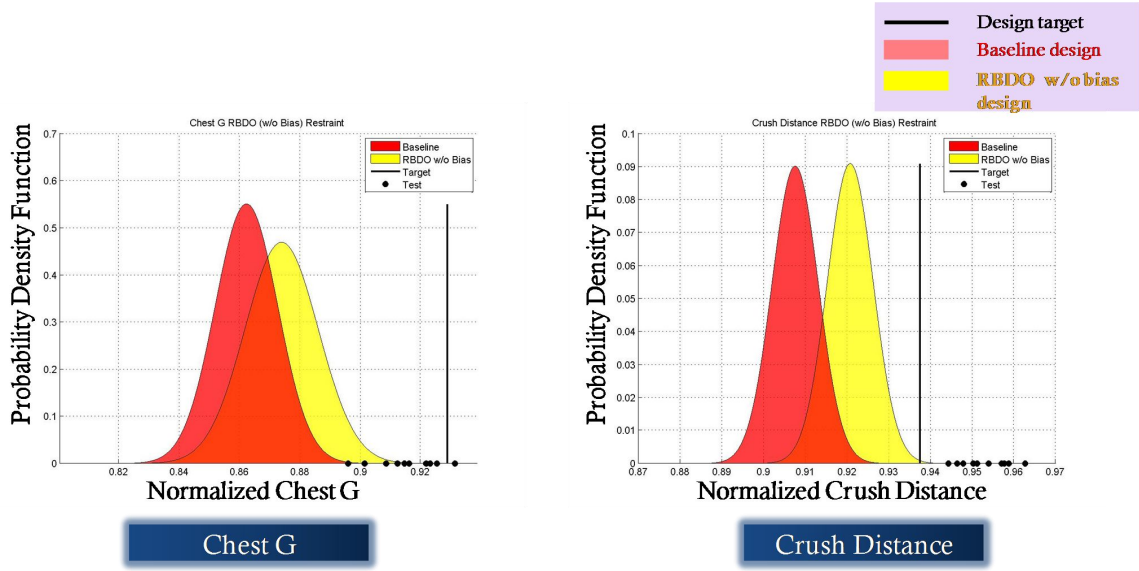


Figure 3 traditional RBDO

On the other hand, Figure shows the FE simulation results for the 10 designs generated from the RBDO solution considering model uncertainty.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Weight
Baseline	1.90	1.91	2.51	2.40	2.55	2.55	2.25	1.50	51.97
RBDO w/o bias	1.715	1.212	1.614	3.966	1.560	1.595	1.559	2.988	44.08
RBDO	1.918	2.270	1.600	1.500	1.601	2.379	1.700	1.497	46.21

All 10 designs satisfy the design targets, confirming the feasibility of the solution. Also, the aggregate probability distributions (from Equation (42)) of Chest G and Crush Distance are also plotted, which are close to the distributions demonstrated from the 10 FE simulations; but the variance of the aggregate distribution is larger than the variance of the FE simulations, due to the model uncertainty we consider.

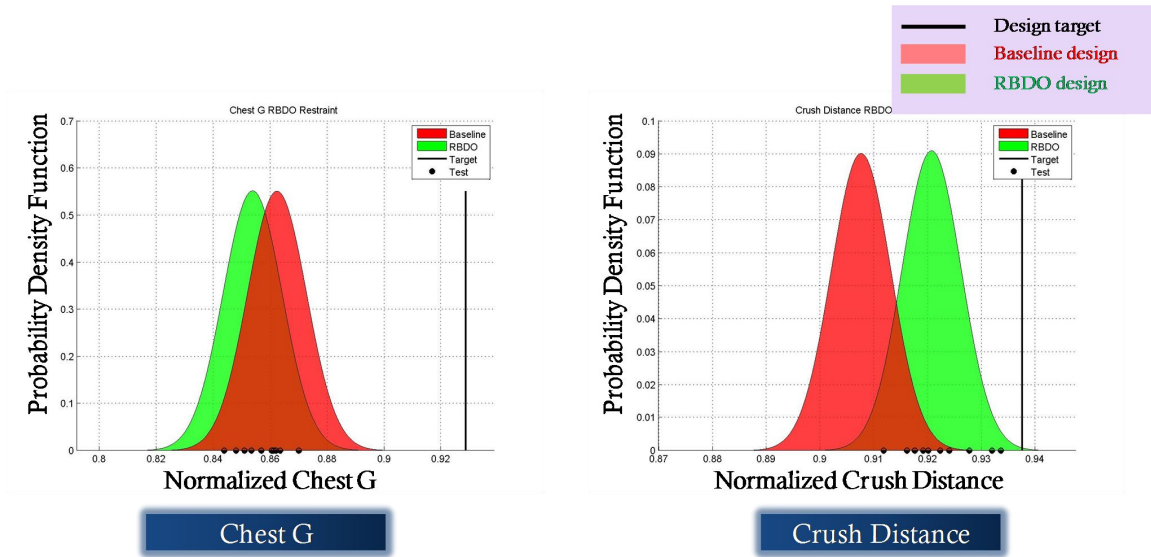


Figure 4 Bayesian inference bias corrected RBDO

This case proves the need for both model bias correction and RBDO considering model uncertainty. The fact that traditional RBDO with a low fidelity CAE model leads to a failed design reminds one to correct the model beforehand and to focus on providing a solution that is reliable not only to the variability of design variables but also to the unknown reality of the true physics.

CONCLUSIONS

This paper proposed to use the Bayesian inference model interpolation and extrapolation method for engineering design. A stochastic bias corrected model extrapolation and robustness design considering model uncertainty and parametric uncertainty is introduced. The Bayesian inference method is first conducted to correct the bias of RSM model and to quantify the model uncertainty. An improved RBDO formulation is then presented to ensure the reliability of optimal design. The proposed approach is demonstrated through a vehicle safety design problem for weight reduction while satisfying safety constraints on Chest G and Crush Distance. Interpolated and extrapolated settings of model inputs validate the proposed model bias correction process. By comparing with the solution from traditional RBDO, the improved RBDO reached a much more reliable solution while achieving the weight reduction.

REFERENCES

1. Oberkampf, W. L., and Roy C. J., 2010, "Verification and Validation in Scientific Computing," Cambridge University Press.
2. Ferson, S., Oberkampf, W. L., and Ginzburg, L., 2008, "Model validation and predictive capability for the thermal challenge problem," Computer Methods in Applied Mechanics and Engineering, 197, No. 29-32, pp. 2408-2430.

3. Schwer, L. E., 2007, "Validation metrics for response histories: perspectives and case studies," *Engineering with Computers*, 23, No.4, pp. 295–309.
4. Fu, Y., Zhan, Z., and Yang, R. J., 2010, "A study of model validation method for dynamic systems," SAE 2011-01-0245, Detroit, MI, April 12-15.
5. Viana, F. A. C., Haftka, R. T., and Steffen, V., Jr., "Multiple Surrogates: How Cross-Validation Errors Can Help Us to Obtain the Best Predictor," *Structural and Multidisciplinary Optimization*, Vol. 39, No. 4, 2009, pp. 439–457. doi:10.1007/s00158-008-0338-0
6. Simpson TW, Peplinski JD, Koch PN, Allen JK (2001) Metamodels for computer-based engineering design: survey and recommendations. *Eng Comput* 17(2):129-150.
7. Wang GG, Shan S (2007) Review of Metamodeling Techniques in Support of Engineering Design Optimization. *J Mech Design* 129(4):370-380.
8. Jin R, Chen W, Simpson TW (2001) Comparative studies of metamodeling techniques under multiple modelling criteria. *Struct Multidisc Optim* 23(1):1-13.
9. Zhang, S., Zhu, P., Chen, W., Arendt, P., "Concurrent treatment of parametric uncertainty and metamodeling uncertainty in robust design," *Structural and multidisciplinary optimization*, 2012, DOI: 10.1007/s00158-012-0805-5.
10. Helton, J. C., Johnson, J. D., and Oberkampf, W. L., 2004, "An Exploration of Alternative Approaches to the Representation of Uncertainty in Model Predictions," *Reliability Engineering and System Safety*, 85, No. 1–3, pp. 39–71.
11. Arendt, P. D., Chen, W., and Apley, D. W., 2011, "Improving Identifiability in Model Calibration Using Multiple Responses." DETC2011-48623, Washington DC, August 28-31, 2011.
12. Mahadevan, S., and Rebba, R., 2005, "Validation of reliability computational models using Bayes networks," *Reliability Engineering and System Safety*, 87, No. 2, pp. 223–232.
13. Rebba, R., and Mahadevan, S., 2006, "Model predictive capability assessment under uncertainty," *AIAA Journal*, 44, No. 10, pp. 2376–2384.
14. Jiang, X., and Mahadevan, S., 2007, "Bayesian risk-based decision method for model validation under uncertainty," *Reliability Engineering and System Safety*, 92, No. 6, pp. 707–718.
15. Chen, W., Baghdasaryan, L., Buranathiti, T., and Cao, J., "Model Validation via Uncertainty Propagation and Data Transformations", *AIAA Journal*, 42(7), 1406-1415, 2004.
16. Zhan, Z., Fu, Y., Yang, R. J., On Stochastic Model Interpolation and Extrapolation Methods for Vehicle Design, *SAE International Journal of Material and Manufacturing*, 6(3):517-531, 2013, doi:10.4271/2013-01-1386.
17. Zhan, Z., Fu, Y., Yang, R. J., Xi, Z., Shi, L., A Bayesian Inference based Model Interpolation and Extrapolation, *SAE International Journal of Material and Manufacturing*, Vol. 5, No. 2, 2012-01-0223, 2012.
18. Zhan, Z., Fu, Y., Yang, R. J., A Research on Stochastic Bias Corrected Model Extrapolation and Robustness Design, SAE 2014 World Congress, SAE2014IDM-0054, under reviewing.
19. Jiang, Z, Chen, W, Fu, Y, Yang, R.J., Reliability-Based Design Optimization with Model Bias and Data Uncertainty, SAE International 2013 technical paper, under review.
20. Xi, Z, Fu, Y., Yang, R.-J., A Model Bias Prediction System for Reliability Based Design, SAE International 2013 technical paper, under review.
21. Jiang, X., Yang, R. J., Barbat, S., Weerappuli, P., Bayesian probabilistic PCA approach for model validation of dynamic systems[J], *SAE International Journal of Materials & Manufacturing*. 2009, 2(1): 555-563.
22. Zhan, Z., Fu, Y. and Yang, R. J., 2011, 'An Enhanced Bayesian Based Model Validation Method for Dynamic Systems', *Journal of Mechanical Design*, 133, No. 4, pp. 041005-1-041005-7.
23. Zhang, R. and Mahadevan, S. (2001) 'Integration of computation and testing for reliability estimation', *Reliability Engineering & System*, Vol. 74, pp. 13–21.
24. Zhan, Z., Fu, Y., Yang, R. J., Peng, Y., "Development and Application of a Reliability-Based Multivariate Model Validation Method", *International Journal of Vehicle Design*, Vol. 60, Nos. 3/4, 194-205, 2012.
25. Liu, Y., Chen, W., Arendt, P. D., and Huang, H. Z., 2011, "Towards a better understanding of model validation metrics," *Journal of Mechanical Design*, 133, No.7, pp. 071005-1-071005-13.
26. Forrester A. I. J., Sobester A., Keane A. J., (2007) Multi-fidelity optimization via surrogate modeling. *P Roy Soc A-Math Phys* 463:3251-3269.
27. National Crash Analysis Center (NCAC) , 2011, www.ncac.gwu.edu/archives/model/index.html.

28. Shi, L., Yang, R. J., Zhu, P., 2012, "An Adaptive Response Surface Method for Crashworthiness Optimization," International Journal of Vehicle Design, to app