面向设计优化的输入不确定性分析方法及其在汽车设计中的应用*

An Input Uncertainty Representation Method for Design Optimization and its Application in Vehicle Design*

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AbstractThis paper develops an enhanced method to represent the input uncertainty for response surface models based design optimization of automotive applications. In the proposed framework, a novel Bootstrap sampling method is firstly proposed to handle the lack of input information. Secondly, the re-sampled data are used as initial data in the process of assessing the effect of input variation on simulation output. Then, the distributions of resampled data are integrated gained in the efficient assessment method for input uncertainty. Commercial available optimizationplatform modeFRONTIER is employed in this study. A real-world vehicle design problem areused to illustrate the proposed procedure and demonstrate the validity of the method.

Key words: Input Representation, Design Optimization, Uncertainty Analysis, Response Surface Model

1Introduction

Vehicle lightweight is a well-known strategy to address growing concerns about harmful emissions, improve fuel economy and save energy. Since the body structure possesses about 40% weight of full vehicle, the body structure lightweight plays a quite important role in weight reduction of full vehicle.

Body structure lightweight design is a systematical project which is involved with crashworthiness, stiffness, modal and noise, vibration, harshness performance. In practice, integrating structural optimization directly with expensive FEsimulations is generally infeasible since optimization search typically requires thousands or even millions of simulations on crashworthiness. As a consequence, there is a growing interest in using RSM to approximate the complicated highly nonlinear behaviors to manage the complexity in analysis and optimization for body weight reduction.The RSM is drafted combining the design of experiments, simulation experiments, and analysis of data by this method. Expression of parameters can be established between input and output with RSM.

However, RSM introduces additional sources of uncertainty, such as model bias, which largely affect the reliability and robustness of the prediction results[1]. The bias of RSM need to be addressed before the model is ready for design optimization.Zhan[2]introduced a Bayesian based RSM correction method and area metric for model validation.For the purpose of correcting the bias in RSMs, scheduling Design of

Experiments (DOEs) must be conducted properly.

Ideal DOEs for correcting bias of RSM should take the effect of input uncertainty on output into consideration. For the purpose of studying the statistical characteristics of discrete points in DOEs, it is critical to represent the input under uncertainty in appropriate way. Plenty of researchers contributed a lot in the investigation of representing the input uncertainty. Probability theory intended only for aleatory uncertainty is a frequently used method for representing such kind of uncertainty but not an appropriate way for epistemic uncertainty[3]. Several theories aiming at handling the type of epistemic uncertainty have been developed. Zadeh[4] proposed fuzzy set theory for various research field under imprecise information conditions.Evidence theory[5,6,7,8,9,10] had been investigated to propose an approach for uncertainty quantification. Aven provided an idea to interpret several alternative uncertainty representations[11]. But these approaches aimed only at the interval uncertainty. Though Shankar and Sankaran[12] has investigated a representation framework for epistemic uncertainty including both interval and sparse points data and applied such an approach on model validation, they conducted the representation process without the consideration of the influence of input uncertainty on simulation output。To tackle such problem, this paper proposed an enhanced input uncertainty representation method aiming at obtaining response surface models with high fidelity.

The rest of the paper is organized as follows: The next section introduces the framework of input uncertainty representation. In the section, a novel sampling method, entropy based input uncertainty assessment and parametric uncertainty representation for resampled data are included. Then Bayesian inference based method for revising bias of RSM is introduced. And the proposed method is applied to a vehicle weight reduction design case. At last, some conclusions are given in the end.

2 Proposed framework for input uncertainty representation

In this paper, we focus on only one type of epistemic uncertainty, namely discrete samples. Figure 1 illustrates the general steps of the proposed method for representing of input uncertainty. Given a computational model, its input and corresponding output are indicated by X and Y, respectively. Firstly, a novel resampling method is utilized to get new data after adding points into the original one, based on which the statistic characteristics of new samples are obtained. Secondly, the uncertainty of these samples is assessed using the criteria of entropy. Lastly, a group of distributions are integrated into a single distribution with high fidelity and reliability. Then randomly resampling can be conducted based on the integrated distribution. By doing so, DOEs with the consideration of the effect of input uncertainty on output is generated to correct the bias of RSM. Commercial available optimizationplatform modeFRONTIER is employed throughout the study process.Details of the three steps are introduced in the following subsections.

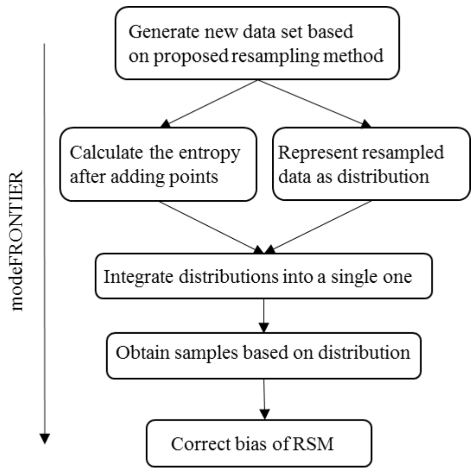

Fig.1 Framework of input uncertainty representation

2.1 A novel sampling method

Normally, response surface is built with discrete data. For the sake of investigating the effect of input on output, it is critical to propose a sampling method. Several theories are developed by excellent researchers. Bootstrap is common in resampling from the original data set to get an estimation of the true distribution, hyper-parameters and so on. With respect to input data, given the original samples $\boldsymbol{X} =$

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$(x_1, x_2 \dots, x_{n-1}, x_n)$ in size of *n*, Bootstrap is randomly sample ***X***for *b* times with the action of replacement. However, we need to modify the data intentionally for investigating how the process effect the output. A novel sampling method is proposed. The difference between our method and Bootstrap is that the novel one adds elements intentionally in a decided schedule while the original one conduct the process randomly. The novel resampling method can be illustrated as add-***L*** explicitly. A set $\boldsymbol{L} = (x_i, \dots, x_{i+j})$in the length of ***L*** is picked out in the input set$\boldsymbol{X} = (x_1, x_2, \dots, x_{n-1}, x_n)$, *i*=(1 to *n*-*l*+1), ***l*** depends on the original size of ***X***. The following step is to add ***L*** to ***X*** several times, the new set is denoted by $\boldsymbol{X}_{a\_l} = (L, \dots, L, x_{i+j+1}, \dots, x_n)$.

2.2 Entropy based input uncertainty assessment method

The additional uncertainty caused by the process of adding new points needs to be assessed. In this section, criteria of entropy is defined as the measurement of uncertainty of random variable. For a discrete random variable X with probability distribution:

$$P(X = x_i) = p_i\,, \quad i = 1,2,3, \dots, n$$

The entropy of X is represented as:

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i \tag{1}$$

The bigger the entropy, the greater the uncertainty of variable. Then, a ratio $\omega$is introduced as weight factor for distributions generated with resampled data sets.

$$\omega = \frac{H(X_i)}{H(X_1) + H(X_2) + \cdots + H(X_n)} \quad i = 1,2, \dots, n \tag{2}$$

The weight factors are beneficial to integrate several input distributions into a single one with fidelity, which will be introduced in the following.

2.3Integrated parametric probability distribution for resampled data

After obtaining the data sets with the proposed sampling method, probability distribution is utilized to represent the uncertainty. Mahadevan and Rebba[13] introduced a methodology to include different sources of uncertainty, including interval inputs, distributions and sparse points. In parametric way, it is necessary to assume the type of distribution (normal, exponential, etc.) of $\boldsymbol{X}_{a\_l}(b)$, *b=1 to n-l+1*. P denotes the hyper-parameters (i.e. mean and standard deviation) of the assumed distribution. The prior probability density function (PDF) of the sample in the condition of parameter P is denoted by$f_{x(x|p)}$. It is critical to estimate P using the efficient method since this prior distribution depends upon the choice of P. The expression of likelihood for P can be derived from the first principles of mathematical statistics. Consider a single point x_i in data set $\boldsymbol{X}_{a\_l}$ that resampled from original data X before, as Shankar and Sanka[12,14] illustrated, we can get the likelihood of this single observation x_i with the help of a constantε. Pawitan[15] has defined the notion of likelihood as the probability of x_i given P. Under the assumption of a small enough and arbitrary constant ε, $x_i$can be expressed as an infinitesimally interval$[\mathrm{x} - \varepsilon/2\,, \mathrm{x} + \varepsilon/2]$. So the likelihood of P is

$$L(p) = p(x \in (x - \varepsilon/2\,, x + \varepsilon/2)|p) = \int_{x-\varepsilon/2}^{x+\varepsilon/2} f_x(x_{i|p})dx \tag{3}$$

According to the mean value theorem of integration, it can be transferred to

$$L(p) = p(x \in (x - \varepsilon/2\,, x + \varepsilon/2)|p) = \varepsilon f_x(x_i|P) \tag{4}$$

For multiple (i.e. n+1) points in the set, the overall likelihood is calculated as

$$L(P) \propto \prod_{i=1}^{n+b.l} f_x(x_i|p) \tag{5}$$

The observing points are all independent in this paper if without intentional note. In the theory of maximum likelihood estimation, after obtaining the likelihood function, parameter P can be estimated by maximizing the function in Equation 5.The distribution of X with parameter P, namely $f_x(x)$ is calculated for every sample of$\boldsymbol{X}_{a_l}$. Combining with the weight factors for each sample evaluated, those distributions can be integrated into a single one as follow:

$$f(x) = \omega_1 * fx(x)_1 + \cdots + \omega_{n-l+1} * fx(x)_{n-l+1} \tag{6}$$

Where $\omega_1 \dots \omega_{n-l+1}$are the weight factors assessed. Here, the single distribution with more fidelity is represented by the proposed framework.

2.4 Bayesian inference based method for revising bias

In the procedure of revising bias, Bayesian inference based method is utilized, difference between CAEand test is considered as evidence of prior distribution of bias. From Bayes' theorem, the posterior distribution is equal to the product of the likelihood function and prior$p(\theta)$, normalized (divided) by the probability of the data $p(x)$:

$$p(\theta|x) = \frac{p(x|\theta)p(\theta)}{\int p(x|\theta)p(\theta)d\theta} \quad (7)$$

Given the assumption of normally distributed conjugate prior, let n denote the number of test observations, the posterior distribution of τ_{μ} can be expressed as$\tau_{\mu 1} \sim N(\mu_1, \sigma_1^2)$.Where$\mu_1 = (\frac{\mu_0}{\sigma_0^2} + \frac{\sum_{i=1}^{n} x_i}{\sigma_0^2})/(\frac{1}{\sigma_0^2} + \frac{n}{\sigma_0^2})$, $\sigma_1^2 = (\frac{1}{\sigma_0^2} + \frac{n}{\sigma_0^2})^{-1}$.Based on the test and CAE data, the bias distribution at each design point x is obtained as: $\tau(x) \sim N(\tau_{\mu 1}(x), \tau_{\sigma 2}(x))$.

3A vehicle design optimization case study

The proposed method is also applied on a weight reduction design problem of a car based on RSM. The goal of vehicle design optimization is to reduce the BIW weight under the precondition of satisfying the intended safety performance. In this case, we only focus on motor compartment lower rail, as is shown in Fig.2. By altering value of parameters of this component usingmodeFRONTIER, specified keyword files for dynamic analysis are generated. The structure and design variables are shown in Fig.3. Responses include Chest G, door_left_disp, rocker_left_acce and total mass, etc.

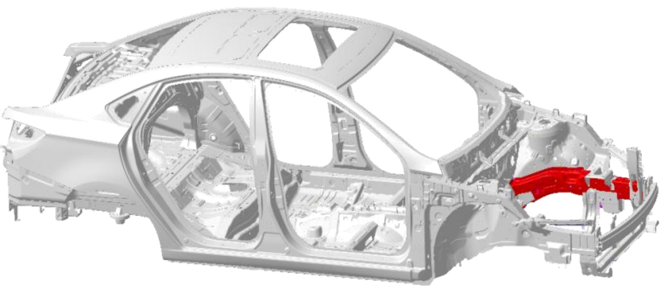


Fig.2 Body in white

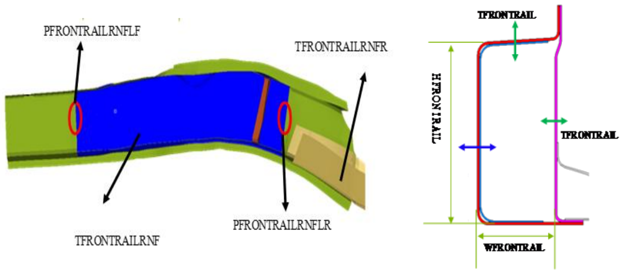


Fig.3Appointed component and design

Details of design variables and responses are provided in Tab.1.Eight design variables are researched. In this paper, only two design variables (i.e. HFRONTRAIL and PFRONTRAILRNFRF) and corresponding one output (i.e. Chest G) are presented for interpretation and application of proposed method. The process of designing the DOE for those two parameters is the same as section 2.1.Firstly, distributions for each variable in Fig.4 are computed with original points based on the introduced method. Secondly, according to distributions, sampling is conducted to gain new DOE for the procedure of building and correcting RSM in Fig.5. The process of building RSM is presented in Fig.6.Finally, area metric is applied for the comparison between new DOE and original uniform design to confirm the advantage of developed methodology.

Tab.1 Design variables and responses list

Design Variable	Baseline	Range		Response
HFRONTRAIL	110	(-1，3)		chest_G
WFRONTRAIL	60	-0.2		door_left_disp
TFRONTRAIL	1.8	(-3,-2,0,2,4)		door_right_disp

PFRONTRAILRNFLF	0.447	(-1,12)	front_wall_disp1
PFRONTRAILRNFLR	0.535	(-7,5)	front_wall_disp2
PFRONTRAILRNFRF	0.765	(-10,5)	rocker_left_acce
PFRONTRAILRNFRR	0.535	(-7,4)	rocker_right_acce
TFRONTRAILRNF	1.2	(-4,-3,-2,0,2,3,4,6)	vel_zero_time_left
TFRONTRAILRNFR	2	(-5,-4,-2,0,2,,4,5)	vel_zero_time_right

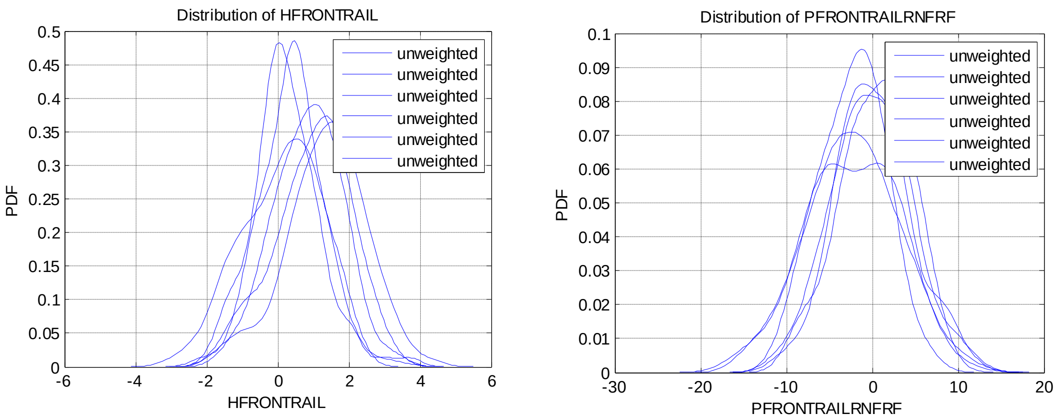


Fig.4 Distributions of HFRONTRAIL and PFRONTRAILRNFRF

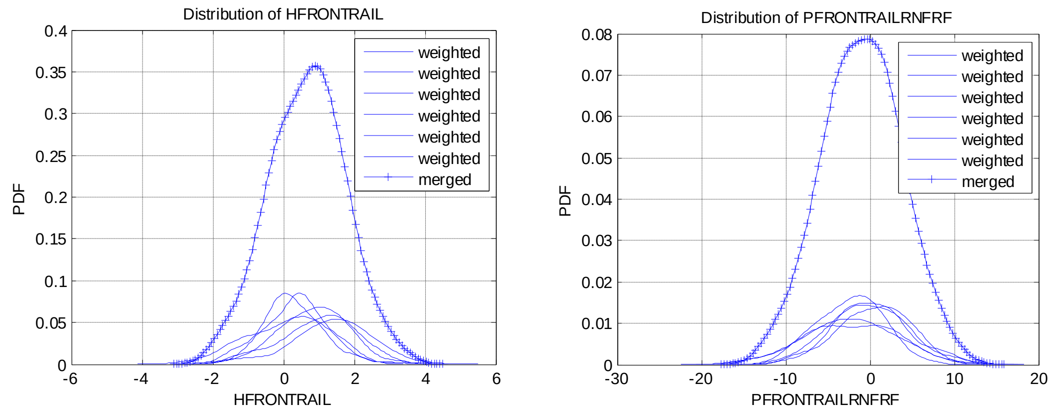


Fig.5 Distributions of HFRONTRAIL and PFRONTRAILRNFRF

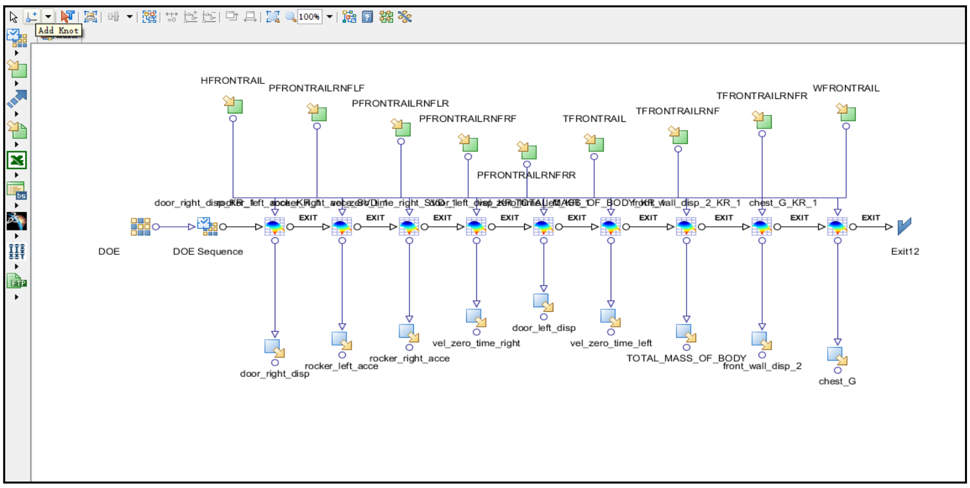
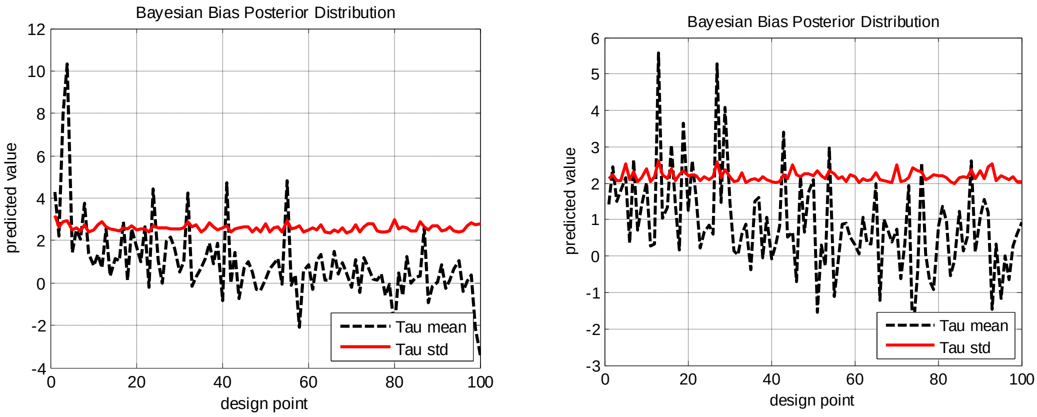


Fig.6 Building process of RSM

100 samples are re-generated for constructing and revising the approximation model for those two method in session 2. With Bayesian theory in Equation 8, posterior bias showed in Figure 7 is calculated to correct the original RSM.Fig.8 shows the flowchart generating bias RSM. Original RSM and the correctedRSM of Chest G corresponding to HFRONTRAIL and PFRONTRAILRNFRF are shown in Fig.9.

$$d\left(f(Y_t), f(Y_{m\_corr})\right) = \int_{-\infty}^{+\infty} \left| f\left(Y_t(x)\right) - f\left(Y_{m_{corr}}(x)\right) \right| dx \quad (8)$$



(a) (b)

Fig.7Posterior bias: (a) uniform sampling (b) distributional sampling

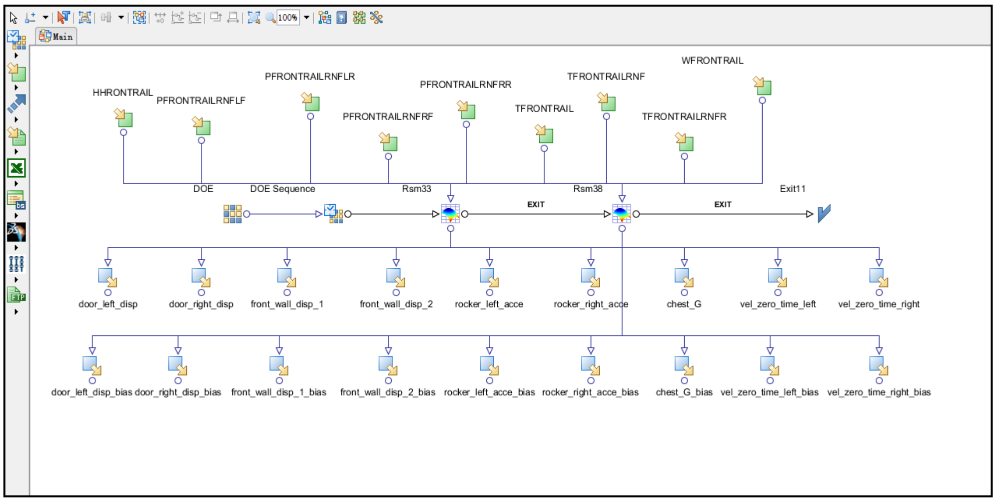
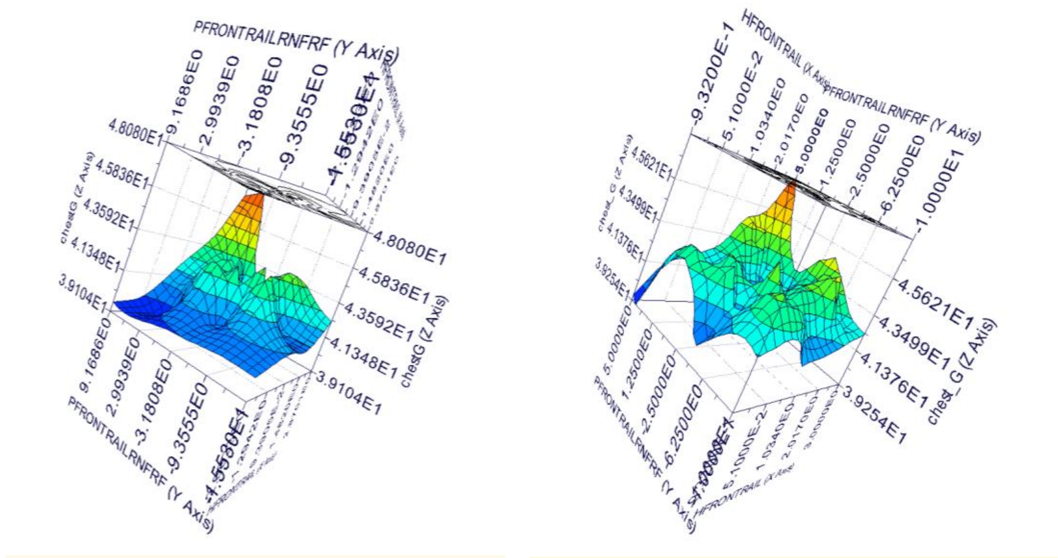


Fig.8 RSM of bias generating



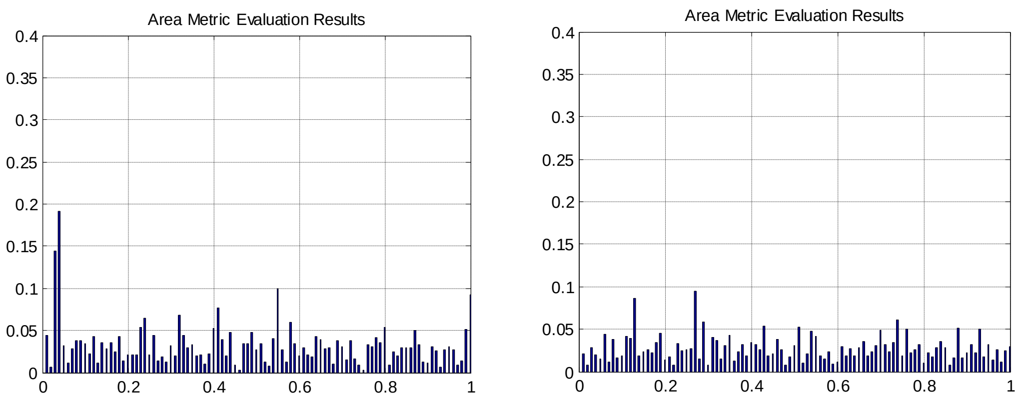
(a) (b)

Fig.9 RSM of Chest G:(a) original RSM,(b) corrected RSM

The results of the comparison based on area metric whose quantitative measure is formulated in Equation 8.and indicated in Fig.10. The quantities of area metric are 0.0329 and 0.0279 according to uniform and distribution based method respectively. The smaller the quantity is, the smaller the discrepancy is. In keeping with our hypothesis, such a process of constructing and revising RSM based on the proposed input representation show better quality in the reduction of bias between RSM and FE model.

Tab.2Quantities of error

Sampling Method	Uniform Sampling	Distributional Sampling
Area Metric(Error)	0.033	0.028



(a) (b)

Fig.10 Area metric: (a) uniform sampling data (b) resampled data by proposed method

The smaller the quantity is, the smaller the discrepancy is. After calculating, the quantities of area

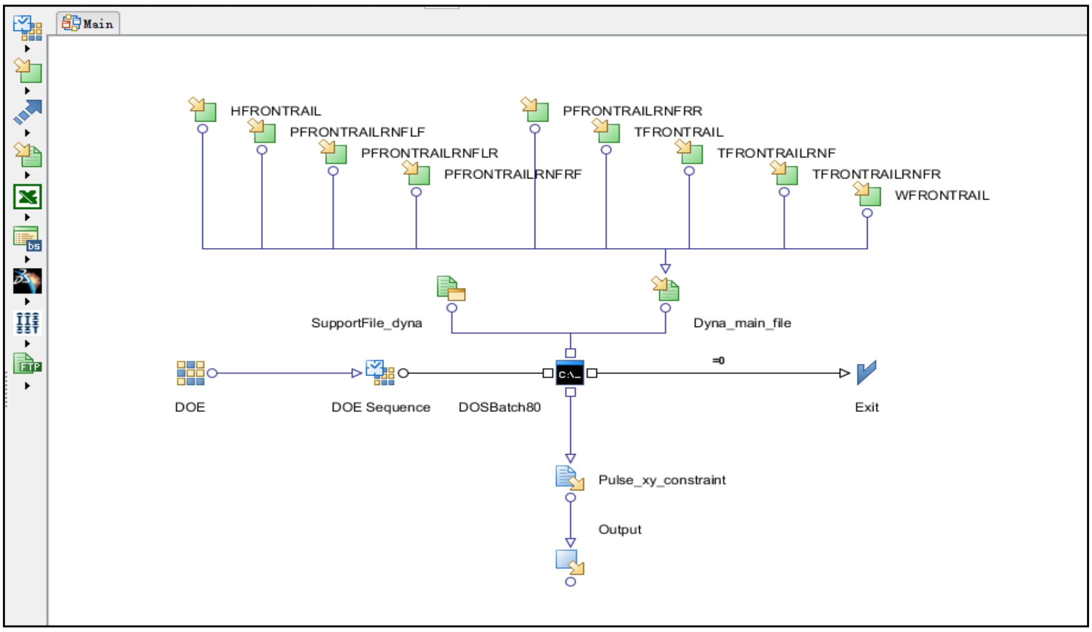


Fig.10 Validation process

metric are 0.0342 and 0.0282 according to uniform and distribution based method respectively, which is presented in Table 2. In keeping with our hypothesis, such a process of constructing and revising RSM based on the pro-posed input representation show better quality in the reduction of bias between RSM and FE model. Finaly,a validation process in Fig.10is applied to verify whether or not the solutions truly meet the safety performance.

4Conclusions

This paper proposed a framework for representing the input epistemic uncertainty. This approach provided the process of how to obtain a single distribution with the poor prior information of input data and assumption.

The input is provided in the form of sparse data which is known as a kind of epistemic uncertainty. When conduct uncertainty analysis (i.e. uncertainty propagation), this kind of poor points are obviously not enough for the following analysis. The proposed methodology provided a novel way to represent the input data in the form of distribution. In the approach, a novel Bootstrap was proposed to get the resamples for the judgment of the effect of input uncertainty on simulation output. Finally, a single distribution was obtained according to the weight factors got in the efficient assessment method. With the illustration and analysis of numerical example in this paper, conclusion can be made that the proposed framework has the ability to handle such a problem in condition of sparse points of input in fidelity and the obtained distribution contains the more characteristic of prior information with high reliability. Also, it was implemented on a more complex engineering case on the auto safety application and vehicle design optimization of BIW weightreduction to validate the fidelity and reliability.

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